Length of a Circular Arc

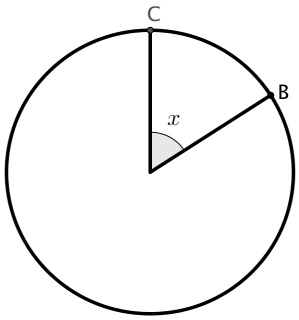
Arcs have two properties. They have a measurable curvature based upon the corresponding central angle (measure of arc = measure of central angle). Arcs also have a length as a portion of the circumference.

$$\frac{portion\ \ of\ \ circle}{whole\ \ circle} = \frac{central\ \ angle\ \ in\ \ \text{deg}rees}{360°} = \frac{central\ \ angle\ \ in\ \ radians}{2\pi} = \frac{arc\ \ length}{circumference}$$

$$\frac{x°}{360°} = \frac{length\ \ \overset{\frown}{CB}}{2\pi r}$$

-or -

$$\frac{x\ \ (radians)}{2\pi} = \frac{length\ \ \overset{\frown}{CB}}{2\pi r}$$



Remember:

- circumference of a circle = 2πr

For a central angle **θ** in radians, and arc length **s** - the proportion can be simplified to a formula:

$$\frac{\theta}{2\pi} = \frac{s}{2\pi r}$$
$$s2\pi = \theta 2\pi r$$
$$s = \theta r$$

Length of an Arc: $s = r\theta$
for θ in radians

Examples:

1) For a central angle of π/6 in a circle of radius 10 cm, find the length of the intercepted arc.

2) For a central angle of 4π/7 in a circle of radius 8 in, find the length of the intercepted arc.

3) For a central angle of 40° in a circle of radius 6 cm, find the length of the intercepted arc.

4.) Find the degree measure to the nearest tenth of the central angle in a circle that has an arc length of 87 and a radius of 16 cm.

Area of a Sector

Sector of a circle: a region bounded by a central angle and the intercepted arc

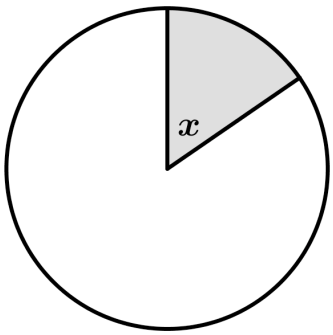
Sectors have an area as a portion of the total area of the circle.

$$\frac{portion\ \ of\ \ circle}{whole\ \ circle} = \frac{central\ \ angle\ \ in\ \ \text{deg}rees}{360°} = \frac{central\ \ angle\ \ in\ \ radians}{2\pi} = \frac{area\ \ of\ \ \text{sec}tor}{area\ \ of\ \ circle}$$

$$\frac{x°}{360°} = \frac{area\ \ of\ \ \text{sec}tor}{\pi r^2}$$

-or -

$$\frac{x\ \ (radians)}{2\pi} = \frac{area\ \ of\ \ \text{sec}tor}{\pi r^2}$$



Remember:

- area of a circle = πr^2

For a central angle **θ** in radians, and area of sector **A**, the proportion can be simplified to a formula:

$$\frac{\theta}{2\pi} = \frac{A}{\pi r^2}$$

$$A2\pi = \theta\pi r^2$$

$$A = \frac{1}{2}\theta r^2$$

Area of a Circular Sector: $A=\frac{1}{2}r^2\theta$ for θ in radians

Examples:

5) Find the area of the sector of the circle that has a central angle measure of π/6 and a radius of 14 cm.

6) Find the area of the sector of the circle that has a central angle measure of 60° and a radius of 9 in.

HONORS

7) A sector has arc length 12 cm and a central angle measuring 1.25 radians Find the radius of the circle and the area of the sector.

Arc & Sector Worksheet

I. Given the radian measure of a central angle, find the length of its intercepted arc in terms of π in a circle of radius 10 cm.

1. $\frac{\pi}{6}$ 2. $\frac{\pi}{3}$ 3. $\frac{\pi}{2}$ 4. $\frac{\pi}{5}$

5. $\frac{3\pi}{5}$ 6. $\frac{4\pi}{7}$ 7. $\frac{\pi}{12}$ 8. $\frac{\pi}{24}$

II. Given the measurement of a central angle, find the measure of its intercepted arc in terms of π in a circle of diameter 60 in.

9. 10° 10. 60° 11. 42° 12. 50°

13. 72° 14. 110° 15. 35° 16. 65°

III. Given the measure of an arc, find the degree measure to the nearest tenth of the central angle if subtends in a circle of radius 16 cm.

17. 87 18. 5.6 19. 12 20. 25

21. 10.24 22. 7.9 23. 11 24. 6

IV. Find the area of each sector to the nearest tenth, given its central angle, and the radius of the circle.

25. $\theta = \frac{\pi}{4}, r = 14 \ \ cm$ 26. $\theta = \frac{\pi}{6}, r = 12 \ \ ft.$ 27. $\theta = \frac{5\pi}{12}, r = 10 \ \ ft.$

28, $\theta = 54°, r = 6 \ \ in$ 29. $\theta = 82°, r = 7.3 \ \ km$ 30. $\theta = 45°, r = 9.75 \ \ mm$

HONORS: (round answers to nearest tenth)

31. A sector has arc length of 6 cm and a central angle measuring 1.2 radians. Find the radius of the circle and the area of the sector.

32. A sector has arc length of 10 in and a central angle measuring 50°. Find the radius of the circle and the area of the sector.