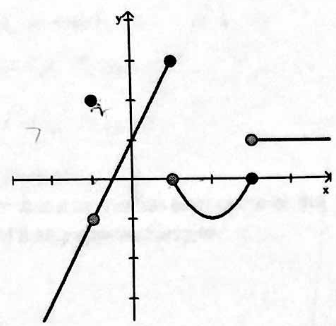
Evaluating Limits – Day 2

Evaluating Limits Graphically – A Review

1. $\lim_{x\to 1^-} f(x)$ 3
2. $\lim_{x\to 0} f(x) = 1$
3. $\lim_{x\to 4} f(x) = 1$
4. $\lim_{x\to 3^+} f(x) = 1$
5. $\lim_{x\to 3^-} f(x) = 0$
6. $\lim_{x\to 3} f(x) =$ DNE
7. $\lim_{x\to -1} f(x) = -1$
8. $\lim_{x\to 1} f(x) =$ DNE



If $f(x) = \begin{cases} x^2+3x+5 & \text{if } x \le -2 \\ 2x+7 & \text{if } x > -2 \end{cases}$

1. $\lim_{x\to -2^-} f(x) = 3$
2. $\lim_{x\to -2^+} f(x) = 3$
3. $\lim_{x\to -2} f(x) = 3$
4. $\lim_{x\to -4} f(x) = 9$
5. $\lim_{x\to 0^-} f(x) = 7$
6. $\lim_{x\to 3} f(x) = 13$

Evaluating Limits Analytically

1. When possible, evaluate a limit using direct substitution!

a. $\lim_{x\to 2} 3x^2 - 5x + 1$

plug in 2!

$\lim_{x\to 2} f(x) = 3(2^2) - 5(2) + 1$

$= \boxed{3}$

b. $\lim_{x\to 6} \frac{5x+2}{x-1}$

$\lim_{x\to 6} f(x) = \frac{5(6)+2}{(6)-1} = \boxed{\frac{32}{5} = 6.4}$

2. If direct substitution yields an undefined answer (#/0) or an indeterminate form (0/0), you must FACTOR and CANCEL to find an equivalent limit!

a. $\lim_{x\to 2} \frac{x^2-4}{x-2}$

$\lim_{x\to 2} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$

$\lim_{x\to 2} (x+2) = 2+2 = \boxed{4}$

b. $\lim_{x\to -3} \frac{x^2+x-6}{x+3}$

$\lim_{x\to 3} \frac{\cancel{(x+3)}(x-2)}{\cancel{(x+3)}}$

$\lim_{x\to 3} (x-2) = \boxed{-5}$

c. $\lim_{x\to 5} \frac{x^2-25}{x^2-4x-5}$

$\lim_{x\to 5} \frac{(x+5)\cancel{(x-5)}}{\cancel{(x-5)}(x+1)}$

$\lim_{x\to 5} \frac{x+5}{x+1} = \frac{10}{6} \boxed{\frac{5}{3}}$

3 Reasons a Limit Does Not Exist

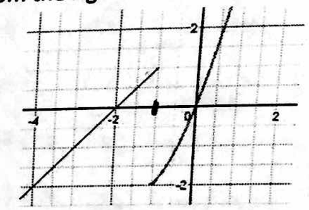
1. The limit approaches a different value from the left and from the right.

$f(x) = \begin{cases} x^2+3x & x > -1 \\ x+2 & x \le -1 \end{cases}$

Find $\lim_{x\to -1} f(x)$ $\boxed{\text{DNE}}$

$\lim_{x\to -1^-} f(x) = 1$

$\lim_{x\to -1^+} f(x) = -2$

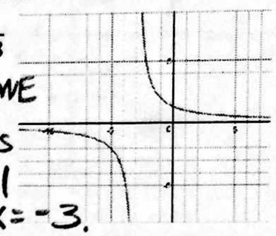


2. The function is unbounded.

$f(x) = \frac{4}{x+3}$

$\lim_{x\to -3} f(x) =$ DNE

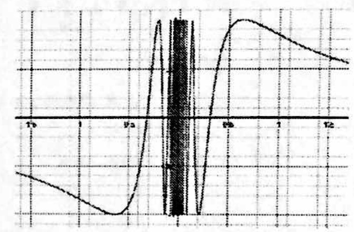
b/c there is a vertical asympt @ $x = -3$.



3. The function oscillates.

$f(x) = \sin(\frac{1}{x})$

$\lim_{x\to 0} f(x) =$ DNE

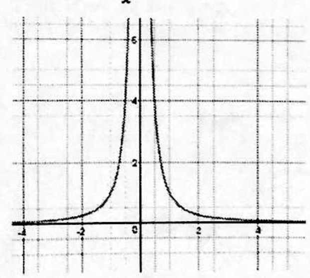


Limits at Vertical Asymptotes:

The line $x = a$ is a vertical asymptote of the function $y = f(x)$ if $f(x)$ approaches a limit of $+\infty$ or $-\infty$ from either direction.

$\lim_{x\to a^+} f(x) = \pm\infty$ or $\lim_{x\to a^-} f(x) = \pm\infty$

Ex. $f(x) = \frac{1}{x^2}$

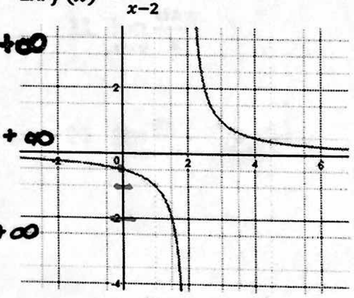


$\lim_{x\to 0^+} f(x) = +\infty$

$\lim_{x\to 0^-} f(x) = +\infty$

$\lim_{x\to 0} f(x) = +\infty$

Ex. $f(x) = \frac{1}{x-2}$



$\lim_{x\to 0^+} f(x) = -\frac{1}{2}$

$\lim_{x\to 0^-} f(x) = -\frac{1}{2}$

$\lim_{x\to 0} f(x) = -\frac{1}{2}$

$\lim_{x\to 2^-} f(x) = -\infty$

$\lim_{x\to 2^+} f(x) = +\infty$

$\lim_{x\to 2} f(x) =$ DNE

... and End Behavior:

Polynomial Functions

Ex. $f(x) = 3x^5 + 3x - 1$

$\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to-\infty} f(x) = -\infty$

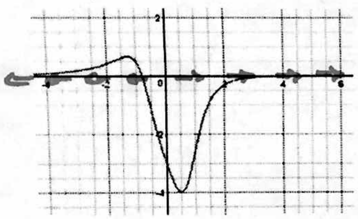
Ex. $f(x) = -4x^2 + 7$

$\lim_{x\to\infty} f(x) = -\infty$

$\lim_{x\to-\infty} f(x) = -\infty$

Rational Functions (Horizontal Asymptotes) The line $y = b$ is a horizontal asymptote of the graph of a function $y = f(x)$ if $f(x)$ approaches a limit of b as x approaches $\pm\infty$.

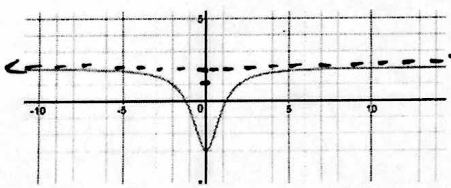
$\lim_{x\to\infty} f(x) = b$ or $\lim_{x\to-\infty} f(x) = b$



Ex. $f(x) = \frac{x^2-3x-3}{x^4+1}$

$\lim_{x\to\infty} f(x) = 0$

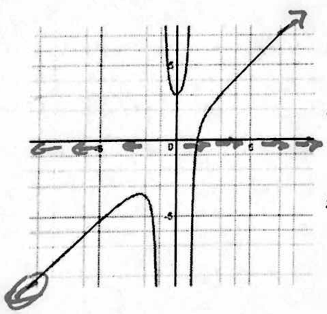
$\lim_{x\to-\infty} f(x) = 0$



Ex. $f(x) = \frac{2x^2-3}{x^2+1}$ HA: $y = \frac{2}{1} = 2$

$\lim_{x\to\infty} f(x) = 2$

$\lim_{x\to-\infty} f(x) = 2$



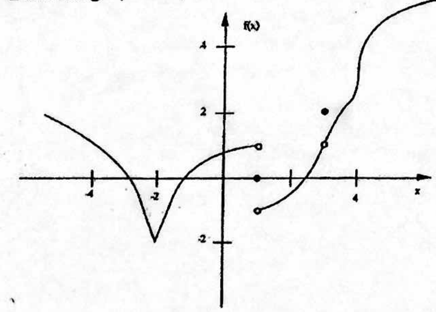
Ex. $f(x) = \frac{x^3-3}{x^2-1}$

$\lim_{x\to\infty} f(x) = +\infty$

$\lim_{x\to-\infty} f(x) = -\infty$

Try these:

A. Evaluate graphically.

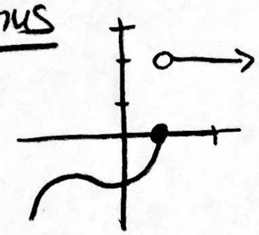


1. $f(1)$ 0
2. $\lim_{x\to1^-} f(x)$ 1
3. $\lim_{x\to1^+} f(x)$ -1
4. $\lim_{x\to1} f(x)$ DNE
5. $f(3)$ 2
6. $\lim_{x\to3^-} f(x)$ 1
7. $\lim_{x\to3^+} f(x)$ 1
8. $\lim_{x\to3} f(x)$ 1
9. $\lim_{x\to-2} f(x)$ -2
10. $\lim_{x\to0} f(x)$ 1
11. $\lim_{x\to-3} f(x)$ 0

B. Find each limit analytically, graphically, or numerically.

12. $\lim_{x\to1} -2x^3 + 5x - 1 = 2$
13. $\lim_{x\to\infty} x^4 + 5x^2$ ∞
14. $\lim_{x\to-\infty} \frac{4x^2+5x}{x^2-1}$ 4
15. $\lim_{x\to2} \frac{2x+7}{x-5}$ $\frac{11}{-3}$
16. $\lim_{x\to5} \frac{2x+7}{x-5}$ DNE
17. $\lim_{x\to\infty} \frac{2x+7}{x-5}$ 2
18. $\lim_{x\to2} \frac{2x+7}{x-5}$ $\frac{11}{-3}$ repeat
19. $\lim_{x\to-2} \frac{2x+4}{x^2-3x-10}$ $\frac{2\cancel{(x+2)}}{(x-5)\cancel{(x+2)}} = \frac{2}{-7}$
20. $\lim_{x\to5} \frac{2x+4}{x^2-3x-10}$ DNE
21. $\lim_{x\to0} \frac{\sin x}{x}$ 1
22. $\lim_{x\to0} \frac{\cos x}{x}$ DNE

Bonus



Explain:

$\lim_{x\to1} f(x) = \text{DNE}$