Standard Normal Calculations   Name: _____
Worksheet #2   Period: _____

1. The EPA fuel economy estimates for automobile models tested recently predicted a Normal model with a mean of 24.8 mpg and a standard deviation of 6.2 mpg. $N(24.8, 6.2)$

a) In what interval would you expect the central 68% of autos to be found?

$(-1z, 1z) =$ from $\boxed{18.6 \text{ mpg to } 31 \text{ mpg}}$ is central 68%

b) About what percent of autos should get less than 31 mpg?

$1 - .16 = .84$ $\boxed{84\%}$ of autos get less than 31 mpg

$N(24.8, 6.2)$ 31

c) What percent of cars should get between 31 and 37 mpg?

$z = \frac{31 - 24.8}{6.2} = 1$ $z = \frac{37 - 24.8}{6.2} = 1.968$

$N(0,1)$ Area = .9726 - .8431
= .1395 or 13.95%

$N(24.8, 6.2)$ 31   37

d) What percent of cars should get more than 20 mpg?

$z = \frac{20 - 24.8}{6.2} = -.774$

Area = $1 - P(z < -.77)$
$= 1 - .2206$
$= .7794$ or $\boxed{77.94\%}$

$N(0,1)$ The worst 20% of cars have mpg of 19.6 mpg or less

e) Describe the gas mileage of the worst 20% of all cars?

z-score for 20th percentile is $-.84$

Find x: $-.84 = \frac{x - 24.8}{6.2}$ $x = 19.592$

Area = .2 $x = ?$ $N(20,16)$

2. Some IQ tests are standardized to a Normal model with a mean of 100 and a standard deviation of 16.

a) What score would begin the interval for the top 16% of all scores? You may use the Empirical Rule to answer this.   @ 1 std. dev. So $\boxed{IQ \text{ of } 116}$

Area = .16 $\bar{x} = \bar{z}$ $N(100,16)$

b) The top 10% of all scores represent the label of "genius". What is the range of scores for anyone who qualifies as a genius?

z-score for 90th percentile: $z = 1.28$

Find x: $1.28 = \frac{x - 100}{16}$ $x = 120.48$

So IQ scores at 120.48 + above is "genius" label.

Area = .10 $\bar{x} = \bar{z}$ $N(100,16)$

c) What proportion of test takers score a 130 or higher?

$z = \frac{130 - 100}{16} = 1.875$

Area = $1 - P(z < 1.875)$
$= 1 - .9695$
$= .0305$ or 3.05% scored 130 or higher on IQ test

$N(0,1)$ 130   1.875

3. Assume the cholesterol levels of Adult American women can be described by a Normal model with a mean of 188 mg/dL and a standard deviation of 24. $N(188, 24)$

a) What percent of adult women do you expect to have cholesterol levels over 200 mg/dL?

$z = \frac{200 - 188}{24} = .5$ $P(z > .5) = 1 - P(z < .5)$
$= 1 - .6915 = .3085$

$N(188, 24)$ $N(0,1)$ 30.85% of women have chol levels over 200 mg/dL

b) What percent of adult women do you expect to have cholesterol levels between 150 and 170 mg/dL?

$z = \frac{150 - 188}{24} = -1.583$
$z = \frac{170 - 188}{24} = -.75$

Area = .2266 - .0571
or .16.95% of women have chol levels between 150 - 170 mg/dL

$N(188, 24)$ $N(0,1)$

c) Estimate the interquartile range of the cholesterol levels.

IQR is 25th to 75th percentile.
$z = -.67$ for 25% $-.67 = \frac{x - 188}{24}$ $x = 171.81$
$z = .67$ for 75% $.67 = \frac{x - 188}{24}$ $x = 204.19$

$\boxed{IQR = 32.39}$

d) Above what value are the highest 15% of women's cholesterol levels?

$z = 1.03$ $1.03 = \frac{x - 188}{24}$ $x = 212.87$

highest 15% of women's cholesterol levels are above 212.87 mg/dL

$N(188, 24)$ $N(0,1)$ Area = .15

4. While only 5% of babies have learned to walk by 13 months of age. If the age at which babies develop the ability to walk can be described by a Normal model, find the mean and standard deviation of the Normal model.

$x = 13$ Area = .05   z-score = 1.645

$\sigma = \frac{13 - \mu}{.675}$ and $\sigma = \frac{13 - \mu}{-1.645}$ solve the system!

$\frac{13 - \mu}{.675} = \frac{13 - \mu}{-1.645}$
$\mu = 12.127$ months $\sigma = 1.29$ months
So $N(12.13, 1.29)$

5. Wildlife biologists believe that the weights of adult trout can be described by a Normal model. Fishermen report that 22% of all trout caught were thrown back because they weigh below the 2-pound minimum, and only 6% weighed over 5 pounds. What are the mean and standard deviation of the model?

Area = .22 $x = 2$ z-score = -.77
Area = .06 $x = 5$ z-score = 1.55

$-.77 = \frac{2 - \mu}{\sigma}$ and $1.55 = \frac{5 - \mu}{\sigma}$

Same as above-system! Find each z-score for given area, put back into σ form

$\frac{5 - \mu}{1.55} = \frac{2 - \mu}{-.77}$

$\mu = 2.996$ lbs $\sigma = 1.293$ lbs
so trout is $N(3, 1.29)$

(think y =), & solve!