A.P. Statistics - Conditional Probability

1. Let a pair of fair dice be tossed. Find the following. Make a sample space.

sum	1	2	3	4	5	6
1	1,1 2	1,2 3	1,3 4	1,4 5	1,5 6	1,6 7
2	2,1 3	2,2 4	2,3 5	2,4 6	2,5 7	2,6 8
3	3,1 4	3,2 5	3,3 6	3,4 7	3,5 8	3,6 9
4	4,1 5	4,2 6	4,3 7	4,4 8	4,5 9	4,6 10
5	5,1 6	5,2 7	5,3 8	5,4 9	5,5 10	5,6 11
6	6,1 7	6,2 8	6,3 9	6,4 10	6,5 11	6,6 12

a) the probability that at least one of the dice is a 4

$P(\text{at least } 1\ 4) = \boxed{\frac{11}{36}}$

b) the probability that the sum is 7

$P(\text{sum } 7) = \boxed{\frac{6}{36}}$

c) Given that the sum is 7, find the probability that one of the dice is 4

$P(\text{at least } 1\ 4 \mid \text{sum } 7) = \dfrac{\frac{2}{36}}{\frac{6}{36}} = \boxed{\frac{1}{3}}$

d) Given that at least one of the dice is 4, find the probability that the sum is 7

$P(\text{sum } 7 \mid \text{at least } 1\ 4) = \dfrac{\frac{2}{36}}{\frac{11}{36}} = \boxed{\frac{2}{11}}$

e) Are rolling two dice with at least one of them a 4 and the sum being 7 a) disjoint b) independent

Explain. (a) No. $P(\text{at least } 1\ 4 \text{ and sum } 7) = \frac{2}{36} \neq 0$ next class!

If they were disjoint $P(\text{at least } 1\ 4 \text{ and sum } 7) = 0$.

2. A cooler has 12 Coke's and 15 Pepsi's. 9 of the Coke's are diet Coke's and 5 of the Pepsi's are diet Pepsi's. A bottle is chosen at random. Find the following: Make a chart. two way table.

	Coke	Pepsi	
Diet	9	5	14
Not Diet	3	10	13
	12	15	27

a) the probability that the bottle is a Coke.

$P(\text{coke}) = \frac{12}{27}$

b) the probability that the bottle is a Pepsi

$P(\text{pepsi}) = \frac{15}{27}$

c) the probability that the bottle is a Diet Coke

$P(\text{diet coke}) = \frac{9}{27}$

d) the probability that the bottle is a Diet Pepsi

$P(\text{diet pepsi}) = \frac{5}{27}$

e) the probability that the bottle is a diet drink

$P(\text{diet}) = \frac{14}{27}$

f) the probability that the bottle is not a diet drink

$P(\text{not diet}) = \frac{13}{27}$

g) Given that the bottle is a diet drink, find the probability that the bottle is a Coke

$P(\text{coke} \mid \text{diet}) = \frac{9}{14}$

h) Given that the bottle is diet drink, find the probability that the bottle is a Pepsi

$P(\text{pepsi} \mid \text{diet}) = \frac{5}{14}$

i) Given that the bottle is a Coke, find the probability that the bottle is a Diet Coke

$P(\text{diet coke} \mid \text{coke}) = \frac{9}{12}$

j) Given that the bottle is a Pepsi, find the probability that the bottle is a Diet Pepsi

$P(\text{diet pepsi} \mid \text{pepsi}) = \frac{5}{15}$

k) Are choosing a regular(non diet) and choosing a diet drink a) disjoint? b) independent? Explain.

(a) $P(\text{non diet and diet}) = 0$ therefore these events are disjoint.

l) Are choosing a Pepsi and choosing a diet drink a) disjoint) b) independent? Explain.

(a) $P(\text{pepsi and diet drink}) = \frac{5}{27}$, so since not equal to zero, these events are not disjoint.

3. A box contains 8 green light bulbs of which 3 are defective. It also contains 12 red light bulbs of which 5 are defective. A bulb is chosen at random from the box. Make a chart.

	red	green	
Defective	5	3	8
Not defective	7	5	12
	12	8	20

a) Find the probability that the bulb is red.

$P(\text{red}) = \frac{12}{20}$

b) If the bulb chosen is red, what is the probability that the bulb is defective?

$P(\text{defective} \mid \text{red}) = \frac{5}{12}$

c) What is the probability that the bulb is defective?

$P(\text{defective}) = \frac{8}{20}$

d) If the bulb is defective, what is the probability that it is red?

$P(\text{red} \mid \text{defective}) = \frac{5}{8}$

e) Is choosing a red bulb and choosing a defective bulb independent? Explain.

4. A woman's club has 80% of its members married. 30% of the married women are pro-choice and 75% of the single women are pro-choice. If a woman is chosen at random, find the probabilities. Make a chart.

	Married	Single	
Pro-choice	24	15	39
Pro-life	56	5	61
	80	20	100

a) she is married

$P(\text{Married}) = 0.8$

b) she is single

$P(\text{single}) = 0.2$

c) she is pro-choice

$P(\text{pro-choice}) = 0.39$

d) she is pro-life

$P(\text{pro-life}) = 0.61$

e) she is married and pro-choice

$P(\text{married} \cap \text{pro-choice}) = 0.24$

f) she is single and pro-choice

$P(\text{single} \cap \text{pro-choice}) = 0.15$

g) Given that she is married, she is pro-choice

$P(\text{pro-choice} \mid \text{married}) = \frac{24}{80}$

h) Given she is married, she is pro-life

$P(\text{pro-life} \mid \text{married}) = \frac{56}{80}$

i) Given she is single, she is pro-choice

$P(\text{pro-choice} \mid \text{single}) = \frac{15}{20}$

j) Given she is single, she is pro-life

$P(\text{pro-life} \mid \text{single}) = \frac{5}{20}$

k) Given she is pro-choice, she is married

$P(\text{Married} \mid \text{pro-choice}) = \frac{24}{39}$

l) Given she is pro-life, she is single

$P(\text{single} \mid \text{pro-life}) = \frac{5}{61}$

m) Are being married and being pro-choice independent? Explain.