Casino Lab – AP Statistics

Casino games are governed by the laws of probability (and those enacted by politicians, too). The same laws (probabilistic, not political) rule the entire known universe. If the universal laws of probability reign in the casino, how come we're always hearing of bettors making a bundle on their hunches? Of solid citizens exalting conjecture over the cosmos. Several factors explain these apparent triumphs of the ridiculous over the sublime. This lab is designed to allow you to analyze some of the games of chance that are typically played in casinos.

STATION 1: CRAPS

Craps is often considered one of the more difficult games to play at casinos. Roll a pair of six-sided dice. If the sum is 7 or 11, you win. If the sum is 2, 3, or 12, you lose. Any other sum is the "point" and if the point is rolled again, you win, but lose on 7. You continue rolling the die until you get the point (WIN!) or roll a 7 (LOSE!).

A. Play 20 games of craps with your partner(s). Each of you should throw the dice for 10 games. Record your results in the tables below. (W = win; L = lose; RA = roll again)

GAME	1ST ROLL	W/L/RA	#ROLLS	FINAL
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

GAME	1ST ROLL	W/L/RA	#ROLLS	FINAL
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

1. In what proportion of the games did you win on your first roll?
2. In what proportion of the games did you win?
3. On average, how many rolls did it take you to win?

B. TI-83 SIMULATION: Using your calculator, you can simulate rolling two dice and obtaining their sum by typing: `randInt(1,6)+randInt(1,6)` and pressing ENTER. Simulate 20 games (10 each) using your calculator. Record your results in the tables on the next page.

GAME	1^{ST} ROLL	W/L/RA	#ROLLS	FINAL
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

GAME	1^{ST} ROLL	W/L/RA	#ROLLS	FINAL
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

C. PROBABILITY QUESTIONS. Use the combined results from your actual games and your calculator simulations.

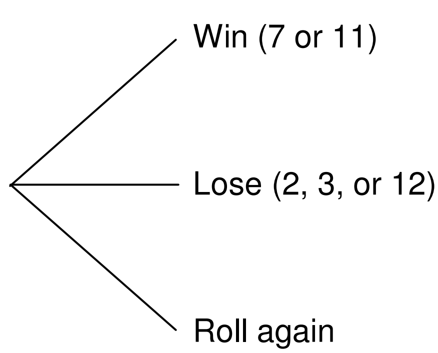
1. What is the probability that you roll a sum of 7 or a sum of 11 on the first roll?

2. What is the probability that you obtain a sum of 2, 3, or 12 on the first roll?

3. What is the probability that you roll again after the first roll?

4. Suppose you roll a sum of 8 on the first roll. Find the probability that you subsequently win the game, given that you rolled an 8 to start with.

D. Tree diagram: Complete the tree diagram shown below for the game of craps.



E. Extension: Find the probability that you win at craps and the expected number of rolls until you get your first win.

STATION 2: ROULETTE

An American roulette wheel has 38 slots, of which 18 are black, 18 are red, and 2 are green. When the wheel is spun, the ball is equally likely to come to rest in any of the slots. This is called "coming in". Players may bet on which number the ball will land on by placing chips on a felt layout. Although the numbers on the wheel itself are in no particular order, the numbers on the felt layout appear in numerical order.

A. Have one partner operate the roulette wheel and let the other partner guess "red" or "black" prior to each spin. Play 20 "games" then switch jobs. Record your results in the table below.

Name	Wins	Losses

B. TI-83 SIMULATION: You can easily simulate the roulette game on your calculator by entering `randInt(1,38)` and letting 37= 0 and 38 = 00. Have one partner operate the calculator and let the other partner guess "red" or "black" prior to each spin. Perform 20 simulations then switch jobs. Record your results in the table below.

Name	Wins	Losses

C. PROBABILITY QUESTIONS. Use the combined results from your actual games and your calculator simulations.

1. Suppose you bet $1 to play a game. If you guess "red" or "black", and are correct, you win $1 and get your original dollar back. If you're wrong, you lose the bet. Let X = the amount gained on a single play. Complete the probability distribution for X.

X		
$P(X)$		

2. Find the expected amount gained per play for an individual player.

3. Calculate the standard deviation of the amount gained per play for an individual player.

4. Find the expected amount gained per play for the house (they get your losses).

5. Suppose there are 50,000 plays on the roulette wheel on any given weekend. How much can the house expect to gain?

D. Another way you can bet is to bet that the ball will land on a certain number, you may bet on any one of the 38 numbers (including 0 and 00). If the ball lands on your number, you will be paid $36 for each dollar you bet.

1. Find the expected amount gained per play for an individual player on this bet.

2. Find the expected amount gained per play for the house (they get your losses).

3. Suppose there are 50,000 plays on the roulette wheel on any given weekend. How much can the house expect to gain?

E. Extension: In Europe, roulette wheels have 18 red and 18 black slots, but only one zero slot. What effect does this have on the calculations you performed in parts (C) and (D)?

STATION 3: BLACKJACK

The game of blackjack begins by dealing 2 cards to a player, the first face-down and the second face-up on top of the first. The player has a “blackjack” if he has a black jack and an ace as his two cards. The player has “twenty one” if he has an ace and a 10, Jack, Queen, or King.

A. Deal 10 blackjack hands, one at a time, shuffling between each hand. That is, deal 2 cards, then check the result, then shuffle, then deal two more cards, etc. Record the number of blackjacks and “twenty-ones” you obtain. Repeat this for your partner.

Name	Blackjacks	Twenty-ones

What proportion of your hands were blackjacks? Twenty-ones?

B. TI-83 SIMULATION: Describe how you would design a simulation to play blackjack on the calculator. Be sure to clearly explain what numbers will represent what cards. Play 5 rounds of blackjack using your model.

What proportion of your hands were blackjacks? Twenty-ones?

C. PROBABILITY QUESTIONS.

1. Given that the face-up card is an ace, find the probability that you have:
   a. a “blackjack” b. twenty-one

2. Given that the face-up card is a black jack, find the probability that you have:
   a. a “blackjack”   b. twenty-one

3. Find the probability of getting a “blackjack” HINT: Consider the sample space of (Top card, Bottom card).

4. Are the events A = face-up card is a black jack and B = you get “blackjack”
   a. independent?   b. disjoint?

   Justify your answers!

STATION 4: MONTE'S DILEMMA

This game is based on the old television show "Let's Make a Deal", hosted by Monte Hall. At the end of each show, the contestant who had won the most money was invited to choose from among three doors: door #1, door #2, or door #3. Behind one of the three doors was a very nice prize, but behind the other two were rather undesirable prizes – say mules. The contestant selected a door. Then, Monte revealed what was behind one of the two doors that the contestant DIDN'T pick (he NEVER showed the prize). He then gave the contestant the option of sticking with the door he had originally selected, or switching.

A. Simulate this game as follows. Pull an ace and two 2s from the deck of cards. These represent the 3 doors with prizes (Ace is the prize). Have your partner arrange the cards and act as a game show host. You pick a "door". Your partner will then show you one of the "doors" you didn't pick (never the Ace). You must then decide to stick with you original choice or to switch "doors." Perform this 10 times and record the results.

Trial	Door chosen	Stick/switch	Win/lose
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

B. Modern Version: Visit the web site http://www.stat.sc.edu/~west/javahtml/LetsMakeaDeal.html . Play 20 games and do NOT switch doors. Record your wins/losses and how many times you won on the first door. Then play 20 games and switch every time. Record your wins/losses and how many times you won on the first door.

	Wins	Losses	First door	% wins
NO Switch				
Switch				

C. PROBABILITY QUESTIONS.

1. What's the probability that picked the door with the nice prize behind it in the first place?

2. Based on the combination of your two simulations, which is better, to stay or switch?

3. Intuition tells us that it shouldn't make any difference whether you stick or switch. There's still a 1/3 chance that you're right. Agree or disagree? Explain.

D. Extension: A woman and man (unrelated) each have two children. At least one of the women's children is a boy, and the man's older child is a boy. Which is more likely: that the man has 2 boys or that the woman has 2 boys?