

More on Errors and Power

1. Your company markets a computerized medical diagnostic program. The program scans the results of medical tests and either clears the patient (they can go home) or refers the case to a doctor. The program is used to screen thousands of people who do not have specific medical complaints and it makes a decision about each person.

a. What are the two hypotheses and the two types of error that the program can make? Try to describe the two types of error in terms of “false positive” and “false negative” test results.

b. The program can be adjusted to decrease one error probability at the cost of an increase in the other error probability. Which error probability would you choose to make smaller, and why?

2. You have the NAEP quantitative scores for an SRS of 840 young Americans. You plan to test hypotheses about the population mean score,

$$H_0: \mu = 275$$

$$H_a: \mu < 275$$

at the 1% level of significance. The population standard deviation is known to be $\sigma = 60$. The z test statistic is

$$z = \frac{\bar{x} - 275}{60/\sqrt{840}}$$

a. What is the rule for rejecting H_0 in terms of z ? (For what values of z would you reject H_0 ?)

b. What is the probability of a Type I error?

3. You are thinking about opening a restaurant and are searching for a good location. From research you have done, you know that the mean income of those living near the restaurant must be over \$45,000 to support the type of upscale restaurant you wish to open. You decide to take an SRS of 50 people living near one potential location. Based on the mean income of this sample, you will decide whether to open a restaurant there. A number of similar studies have shown that $\sigma = \$5,000$.

a. State the null and alternative hypotheses.

b. Describe the two types of errors that you might make.

Type I:

Type II:

c. Which of the two types of error is most serious? Explain.

d. If you had to choose one of the “standard” significant levels for your significance test, would you choose $\alpha = 0.01$, 0.05 , or 0.10 ? Justify your choice.

e. Based on your choice in part (d), how high will the sample mean need to be before you decide to open a restaurant in that area?

4. Statisticians prefer large samples. Describe briefly the effect of increasing the size of a sample (or the number of subjects in an experiment) on each of the following:

a. The margin of error of a 95% confidence interval.

b. The P-value of a test, when H_0 is false and all facts about the population remain unchanged as n increases.

c. The power of a fixed level α test, when α , the alternative hypothesis, and all facts about the population remain unchanged.

5. When asked to explain the meaning of “the P-value was $P = 0.03$,” a student says, “This means there is only probability 0.03 that the null hypothesis is true.” Is this an essentially correct explanation? Explain your answer.