More on Errors and Power

1. Your company markets a computerized medical diagnostic program. The program scans the results of medical tests and either clears the patient (they can go home) or refers the case to a doctor. The program is used to screen thousands of people who do not have specific medical complaints and it makes a decision about each person.

a. What are the two hypotheses and the two types of error that the program can make? Try to describe the two types of error in terms of "false positive" and "false negative" test results.

H_0: the patient can go home

H_a: refer the case to a doctor

Type I: the program thinks the case should go to a doctor, but the patient should go home
↳ false positive

Type II: the program thinks the patient should go home, but the case should go to a doctor
↳ false negative

b. The program can be adjusted to decrease one error probability at the cost of an increase in the other error probability. Which error probability would you choose to make smaller, and why?

It would probably be better to decrease the probability of making a Type II error so that people are receiving the medical attention they need. We would like to reduce the probability of sending sick people home.

2. You have the NAEP quantitative scores for an SRS of 840 young Americans. You plan to test hypotheses about the population mean score,

H_0: $\mu = 275$

H_a: $\mu < 275$

at the 1% level of significance. The population standard deviation is known to be $\sigma = 60$. The z test statistic is

$$z = \frac{\bar{x} - 275}{60/\sqrt{840}}$$

a. What is the rule for rejecting H_0 in terms of z? (For what values of z would you reject H_0?)

$Z < -2.33$ This is the z-score for a left-tail probability of .01 (1%)

b. What is the probability of a Type I error?

.01

3. You are thinking about opening a restaurant and are searching for a good location. From research you have done, you know that the mean income of those living near the restaurant must be over $45,000 to support the type of upscale restaurant you wish to open. You decide to take an SRS of 50 people living near one potential location. Based on the mean income of this sample, you will decide whether to open a restaurant there. A number of similar studies have shown that $\sigma = \$5{,}000$.

a. State the null and alternative hypotheses.

$H_0: \mu = 45{,}000$ the mean income is too low to build at this location

$H_a: \mu > 45{,}000$ the mean income is high enough to build at this location

b. Describe the two types of errors that you might make.

Type I: We think it's safe to build here, but in fact it is not.

Type II: We think it's not safe to build here, but in fact it is.

c. Which of the two types of error is most serious? Explain.

Type I We will build the restaurant in this location but won't get as much business as expected and the restaurant will probably fail.

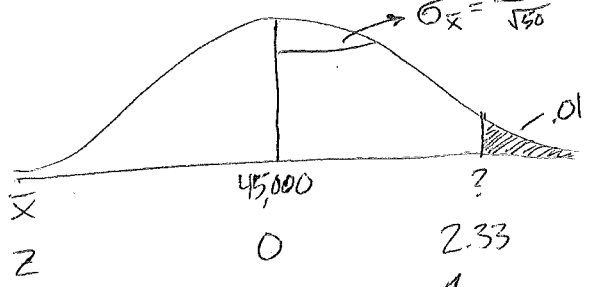
d. If you had to choose one of the "standard" significant levels for your significance test, would you choose $\alpha = 0.01, 0.05$, or 0.10? Justify your choice.

$\alpha = .01$

Since a Type I error was most serious, decreasing the α-level would reduce the chances of making this error.

e. Based on your choice in part (d), how high will the sample mean need to be before you decide to open a restaurant in that area?

$\sigma_{\bar{x}} = \frac{5000}{\sqrt{50}} = 707.11$



This is the z-score for the 99th percentile

$707.11 \times 2.33 \leq \frac{\bar{x} - 45000}{707.11} \times 707.11$

$1647.5663 \leq \bar{x} - 45{,}000$

$+45{,}000 \qquad +45{,}000$

$46{,}647.57 \leq \bar{x}$