

**KEY**

## One-Sample z Tests

1. The National Center for Health Statistics reports that the mean blood pressure for males 35-44 years of age is 128 and the standard deviation in this population is 15. The medical director of 3M looks at the medical records of a random sample of 72 executives in this age group and finds that the mean blood pressure in this sample is  $\bar{x} = 126.07$ . Is there significant evidence at the 5% level that 3M's executives have a different mean blood pressure from the general population? Assume that 3M has over 1000 male executives in this age group.

**P:** State what the parameter of interest is representing in this problem.

$\mu$  = mean blood pressure of all male 3M executives 35-44 years of age.

**H:** State hypotheses in words and symbols.

$H_0: \mu = 128$  Blood pressures of male 3M executives are no different than the general pop.

$H_a: \mu \neq 128$  Male 3M executives have a different mean blood pressure than the general population.

**A:** Verify the assumptions/conditions.

- ✓ Random. A random sample of 72 executives was taken

- ✓ Normal. Approximately Normal by the CLT since  $n \geq 30$  ( $n = 72$ )

- ✓ Independent.  $n \leq \frac{1}{10} N$  There are more than 720 male executives at 3M.

$$72 \leq \frac{1}{10} N$$

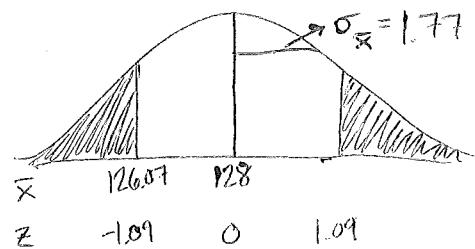
$$720 \leq N$$

**N:** Name the appropriate inference procedure. Since  $\sigma$  is known, we will use a one-sample

**T:** Carry out the selected procedure. Find the test statistic.

$z$  test

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{126.07 - 128}{15 / \sqrt{72}} = -1.09$$



**O:** Obtain the corresponding P-value based on the test statistic and  $H_a$ .

$$\text{P-value} = P(z < -1.09) \times 2 = .2758$$

**M:** Make a decision to reject or fail to reject  $H_0$ .

Because the P-value is not significant at the 5% level, we fail to reject  $H_0$ .

**S:** State your conclusion in the context of the problem.

There is not strong evidence that male 3M executives 35-44 years of age have a different mean blood pressure than the general population.

2. In a discussion of the education level of the American workforce, someone says, "The average young person can't even balance a checkbook." The NAEP survey says that a score of 275 or higher on its test reflects the skill needed to balance a checkbook. The NAEP random sample of 840 young Americans had a mean score of  $\bar{x} = 272$ , a bit below the checkbook-balancing level. If the standard deviation of scores of every young American on the test is  $\sigma = 60$ , is this sample result significant evidence at the 5% level that the mean score for *all* young Americans is less than 275?

P:  $\mu = \text{mean NAEP score of all young Americans}$

H<sub>0</sub>:  $H_0: \mu = 275$  the average young American has a NAEP score that reflects their ability to balance a checkbook

H<sub>a</sub>:  $H_a: \mu < 275$  the average young American can't even balance a checkbook

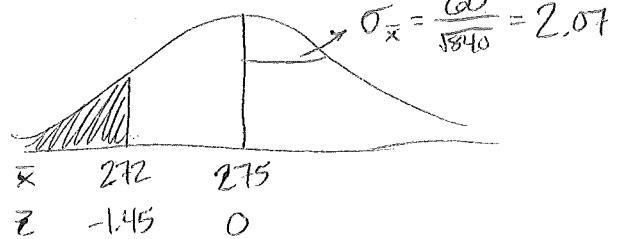
A: ✓Random - A random sample of 840 young Americans was taken

✓Normal - Approximately Normal by the CLT since  $n \geq 30$  ( $n = 840$ )

✓Independent -  $n \leq 10N \rightarrow N \geq 8400$  There are more than 8400 young Americans

N: Since  $\sigma$  is known, we will use a one-sample z test

$$T: z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{272 - 275}{60/\sqrt{840}} = -1.45$$



$$O: P\text{-value} = P(z < -1.45) = .0735$$

M: Since our P-value is not significant at the 5% level, we fail to reject H<sub>0</sub>.

S: There is not strong evidence that the average young American can't balance a checkbook.

3. A pharmaceutical manufacturer forms tablets by compressing a material that contains the active ingredient and various extras. The hardness of a sample from each lot of tablets produced is measured in order to control the compression process. The target values for the hardness are  $\mu = 11.5$  and  $\sigma = 0.2$ . The hardness data for a sample of 20 tablets are

11.627	11.613	11.493	11.602	11.360	11.374	11.592	11.458	11.552	11.463
11.383	11.715	11.485	11.509	11.429	11.477	11.570	11.623	11.472	11.531

Is there significant evidence at the 5% level that the mean hardness of the tablets is different from the target value?

P:  $\mu$  = mean hardness of all tablets produced by this manufacturer

H<sub>0</sub>:  $H_0: \mu = 11.5$  the mean hardness of the tablets meets the target value

H<sub>a</sub>:  $H_a: \mu \neq 11.5$  the mean hardness of the tablets is different than the target value

A: X Random - It is not clear if the tablets were sampled randomly

Since we are not sure if our sample was taken randomly, it is not safe to continue with a one-sample z test. Different sampling methods require different inference procedures.

4. Bottles of Coca-Cola are supposed to contain 300 milliliters (ml) of cola. There is some variation from bottle to bottle because the filling machinery is not perfectly precise. The distribution of the contents is Normal with standard deviation  $\sigma = 3$  ml. An inspector who suspects that the bottler is under filling the bottles measures the contents of six randomly selected bottles. The results are

299.4	297.7	301.0	298.9	300.2	297.0
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$$\bar{x} = 299.03$$

Is this convincing evidence at the 10% level that the mean contents of Coca-Cola bottles is less than the advertised 300 ml?

P:  $H_0: \mu = \text{mean amount of cola in all bottles of Coca-Cola}$

$H_a: H_0: \mu = 300$  mean contents of Coca-Cola bottles is as advertised

$H_a: \mu < 300$  mean contents of Coca-Cola bottles is less than the advertised 300 ml

A: ✓Random - Six bottles were selected randomly

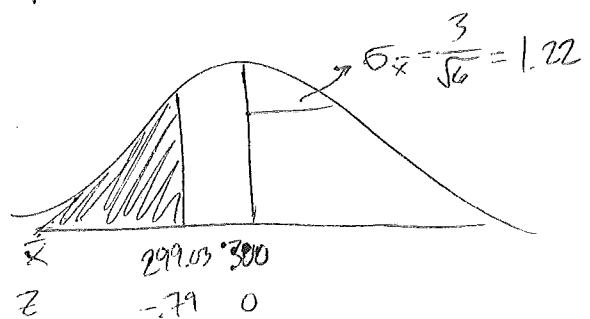
✓Normal - Population is Normal

✓Independent -  $n \leq \frac{1}{10}N \rightarrow N \geq 60$  There are more than 60 bottles of Coca-Cola being filled.

N: Since  $\sigma$  is known, we will use a one-sample Z test.

$$T: Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{299.03 - 300}{3/\sqrt{6}} = -.79$$

$$C: P\text{-value} = P(Z < -.79) = .2148$$



M: Because the P-value is not significant at the  $\alpha = .1$  level, we fail to reject  $H_0$ .

S: There is not strong evidence that the mean contents of Coca-Cola bottles is less than the advertised 300 ml.