

KEY

Paired t Tests

1. Researchers designed an experiment to study the effects of caffeine withdrawal. They recruited 11 volunteers who were diagnosed as being caffeine dependent to serve as subjects. Each subject was barred from coffee, colas, and other substances with caffeine for the duration of the experiment. During one 2-day period, subjects took capsules containing their normal caffeine intake. During another 2-day period, they took placebo capsules. The order in which subjects took caffeine and the placebo was randomized. At the end of each 2-day period, a test for depression was given to all 11 subjects. Researchers wanted to know whether being deprived of caffeine would lead to an increase in depression.

The table below contains data on the subjects' scores on the depression test. Higher scores show more symptoms of depression. For each subject, we calculated the difference in test scores following each of the two treatments (placebo - caffeine). We chose this order of subtraction to get mostly positive values.

Results of a caffeine-deprivation study			
Subject	Depression (caffeine)	Depression (placebo)	Difference (placebo - caffeine)
1	5	16	11
2	5	23	18
3	4	5	1
4	3	7	4
5	8	14	6
6	5	24	19
7	0	6	6
8	0	3	3
9	2	15	13
10	11	12	1
11	1	0	-1

Carry out a significance test at the 1% level to determine if being deprived of caffeine leads to an increase in depression.

P: μ_d = mean difference (placebo - caffeine) in depression scores among all caffeine dependent people.

H: $\mu_d = 0$ Being deprived of caffeine has no effect on depression.

$\mu_d > 0$ Being deprived of caffeine leads to an increase in depression.

A: ☒ Random - The order in which subjects took caffeine and the placebo was randomized.

☒ Normal - A graph of the sample data shows no strong skewness and no outliers.

☒ Independent - Not needed since subjects weren't randomly selected from the population.

N: Paired t test for μ_d

$$T: t = \frac{7.36 - 0}{6.92/\sqrt{11}} = 3.53 \quad df = 11 - 1 = 10$$

$$O: P\text{-value} = P(t > 3.53) = \text{between } .0025 \text{ and } .005$$

= .0027



M: Because the P-value is significant at the 1% level, we reject H_0 .

S: There is strong evidence that being deprived of caffeine leads to an increase in depression.

2. The following table gives the ages of the husband and wife of a simple random sample of couples that have obtained a marriage licenses in Cumberland County, Pennsylvania in 1993.

Couple #	Husband	Wife	H-W	Couple #	Husband	Wife	H-W
1	25	22	3	13	25	24	1
2	25	32	-7	14	23	22	1
3	51	50	1	15	19	16	3
4	25	25	0	16	71	73	-2
5	38	33	5	17	26	27	-1
6	30	27	3	18	31	36	-5
7	60	55	5	19	26	24	2
8	54	57	-3	20	62	60	2
9	31	30	1	21	29	26	3
10	54	47	7	22	31	23	8
11	23	23	0	23	29	28	1
12	34	39	-5	24	35	36	-1

Conduct an appropriate test of significance at the 5% level to determine if there is evidence to support the claim that husbands tend to be older than their wives.

P: μ_d = mean difference (Husband - Wife) in age among all married couples in Cumberland County, PA in 1993

H: $\mu_d = 0$ Husbands and wives are no different in age

$\mu_d > 0$ Husbands tend to be older than their wives

A: \checkmark Random - A simple random sample of couples was obtained.

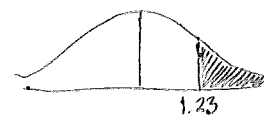
\checkmark Normal - A graph of the sample data shows no strong skewness and no outliers.

\checkmark Independent - $n \leq \frac{1}{10} N \rightarrow N \geq 240$ It's safe to assume this county had more than 240 married couples.

N: Paired t test for μ_d

$$T: t = \frac{.9167 - 0}{3.65/\sqrt{24}} = 1.23 \quad df = 24 - 1 = 23$$

$$O: P\text{-value} = P(t > 1.23) = \text{between .10 and .15} \quad \text{OR} \quad P\text{-value} = .1156$$



M: Because the P-value is not significant at the 5% level, we fail to reject H_0 .

S: There is not strong evidence that husbands in Cumberland County, PA in 1993 tend to be older than their wives.