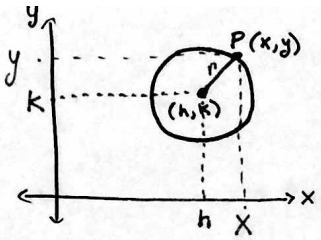
# Equations of Circles

Standard Form of Circles:

$$(x-h)^2 + (y-k)^2 = r^2$$

| center: | radius: | Point on circle: |
|---|---|---|
| $(h,k)$ | $r$ | $(x,y)$ |




Ex: Write eqn. of circle w/ center $(0,0)$ and radius $= 8$.
$(h,k)$ $r$

$$(x-0)^2 + (y-0)^2 = 8^2$$

$$\boxed{x^2 + y^2 = 64}$$

Ex: Write eqn of circle w/ center $(6,-3)$ w/ radius $= 10$.
$(h,k)$ $r$

$$\boxed{(x-6)^2 + (y+3)^2 = 100}$$

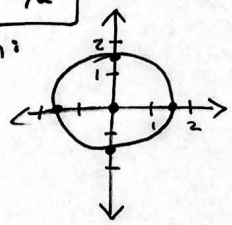
Ex Find the center & radius.

$$x^2 + y^2 = \frac{9}{4}$$

center: $(0,0)$
radius: $3/2$

$\sqrt{r^2} = \sqrt{\frac{9}{4}} = \frac{\sqrt{9}}{\sqrt{4}} = \frac{3}{2}$

Graph:



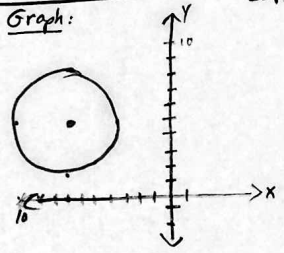
Ex: Find center & radius.

$$(x+7)^2 + (y-5)^2 = 12$$

center: $(-7,5)$
radius: $2\sqrt{3}$

$\sqrt{r^2} = \sqrt{12}$
$= \sqrt{4 \cdot 3}$
$= 2\sqrt{3}$

Graph:



# Write equation of a circle

Ex: Write eqn of circle w/ center $(8,-7)$ and goes through the point $(-3,1)$.
$(8,-7)$ is $(h,k)$ $(-3,1)$ is $(x,y)$

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(-3-8)^2 + (1-{-7})^2 = r^2$$
$$(-11)^2 + (8)^2 = r^2$$
$$121 + 64 = r^2$$
$$185 = r^2$$

Final eqn:

$$(x-8)^2 + (y+7)^2 = 185$$

Ex: Write eqn of circle w/ endpoints of diameter at $(-6,5)$ and $(4,-3)$. $(x,y)$

Need center first! Find midpt:

$$\left(\frac{-6+4}{2}, \frac{5+-3}{2}\right) = \left(\frac{-2}{2}, \frac{2}{2}\right) = (-1,1)$$
$(h,k)$

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(4-{-1})^2 + (-3-1)^2 = r^2$$
$$(5)^2 + (-4)^2 = r^2$$
$$25 + 16 = r^2$$
$$41 = r^2$$

Final eqn:

$$(x+1)^2 + (y-1)^2 = 41$$

Ex: Write eqn of circle w/ center $(h,k) = (0,4)$ and goes through point $(-2,-5)$. $(x,y)$

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(0-{-2})^2 + (4-{-5})^2 = r^2$$
$$(2)^2 + (9)^2 = r^2$$
$$85 = r^2$$

$$\boxed{x^2 + (y-4)^2 = 85}$$

## Write eqn in std. form

Ex: Write eqn in standard form. Then give center & radius.

a) $x^2 + y^2 + 4x - 8y + 16 = 0$.

$$(x^2 + 4x + 4) + (y^2 - 8y + 16) = -16 + 4 + 16$$

$\frac{+4}{2} = (2)^2 = 4$ $\qquad$ $\frac{-8}{2} = (-4)^2 = 16$

$$\boxed{(x+2)^2 + (y-4)^2 = 4}$$

center: $(-2,4)$ radius: $\sqrt{4} = 2$

b) $x^2 + y^2 - 6x - 2y + 4 = 0$

$$(x^2 - 6x + 9) + (y^2 - 2y + 1) = -4 + 9 + 1$$

$$\boxed{(x-3)^2 + (y-1)^2 = 6}$$

center: $(3,1)$ radius: $\sqrt{6}$