Precalculus Name Key

5.1-5.3 Review (no calculator)

Use the given values to evaluate the remaining trigonometric functions of the angle. (no triangles; use identities only)

1. $\cos x = \frac{2}{5}, \tan x < 0$ (Quad. IV)

$\sin^2 x = 1 - \cos^2 x$

$\sin^2 x = 1 - \left(\frac{2}{5}\right)^2$

$\sin^2 x = 1 - \frac{4}{25}$

$\sin^2 x = \frac{21}{25}$ $\sin x = \pm \frac{\sqrt{21}}{5}$

$\sin x = -\frac{\sqrt{21}}{5}$ $\csc x = -\frac{5\sqrt{21}}{21}$

$\cos x = \frac{2}{5}$ $\sec x = \frac{5}{2}$

$\tan x = -\frac{\sqrt{21}}{2}$ $\cot x = -\frac{2\sqrt{21}}{21}$

Simplify the expression.

2. $\frac{1+\csc\theta}{\sec\theta} - \cot\theta$

$\frac{1}{\sec\theta} + \frac{\csc\theta}{\sec\theta} - \frac{\cos\theta}{\sin\theta}$

$\cos\theta + \frac{\cos\theta}{\cancel{\sin\theta}} - \frac{\cos\theta}{\cancel{\sin\theta}}$

$\boxed{\cos\theta}$

3. $\sin^2\theta \cdot \sec^2\theta + \sin^2\theta \cdot \csc^2\theta$

$\sin^2\theta \sec^2\theta + \sin^2\theta \cdot \frac{1}{\sin^2\theta}$

$\sin^2\theta \sec^2\theta + 1$

$\sin^2\theta \left(\frac{1}{\cos^2\theta}\right) + 1$

$\tan^2\theta + 1$

$\boxed{\sec^2\theta}$

Verify each trigonometric identity.

4. $\frac{\csc^2 x}{\cot x} = \csc x \cdot \sec x$

$\csc^2 x \cdot \tan x = \csc x \cdot \sec x$

$\frac{1}{\sin^2 x} \cdot \frac{\sin x}{\cos x} = \csc x \cdot \sec x$

$\frac{1}{\sin x} \cdot \frac{1}{\cos x} = \csc x \cdot \sec x$

$\csc x \cdot \sec x = \csc x \cdot \sec x$ ✓

5. $\cos^2 x - \sin^2 x = 1 - 2\sin^2 x$

$(1-\sin^2 x) - \sin^2 x = 1 - 2\sin^2 x$

$1 - 2\sin^2 x = 1 - 2\sin^2 x$ ✓

6. $\csc\beta - \sin\beta = \cos\beta \cdot \cot\beta$

$\frac{1}{\sin\beta} - \frac{\sin\beta}{1} = \cos\beta \cdot \cot\beta$

$\frac{1}{\sin\beta} - \frac{\sin^2\beta}{\sin\beta} = \frac{\cos\beta}{1} \cdot \frac{\cos\beta}{\sin\beta}$

$\frac{1-\sin^2\beta}{\sin\beta} = \frac{\cos^2\beta}{\sin\beta}$

$\frac{\cos^2\beta}{\sin\beta} = \frac{\cos^2\beta}{\sin\beta}$ ✓

7. $(\tan^2 x + 1)(\cos^2 x - 1) = -\tan^2 x$

$(\sec^2 x)(-\sin^2 x) = -\tan^2 x$

$\left(\frac{1}{\cos^2 x}\right)\left(\frac{-\sin^2 x}{1}\right) = -\tan^2 x$

$-\frac{\sin^2 x}{\cos^2 x} = -\tan^2 x$

$-\tan^2 x = -\tan^2 x$ ✓

Solve in the interval $[0,2\pi)$.

8. $\sin x = \sqrt{3} - \sin x$

$2\sin x = \sqrt{3}$

$\sin x = \frac{\sqrt{3}}{2}$

$\boxed{x = \frac{\pi}{3}, \frac{2\pi}{3}}$

9. $\sec x \cdot \csc x = 2\csc x$

$\sec x \cdot \csc x - 2\csc x = 0$

$\csc x(\sec x - 2) = 0$

$\csc x = 0$ $\quad$ $\sec x - 2 = 0$

$\varnothing$ $\quad$ $\sec x = 2$

$\cos x = \frac{1}{2}$ $\quad$ $\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$

10. $\cos^2 x + \sin x = 1$

$(1-\sin^2 x) + \sin x = 1$

$1 - \sin^2 x + \sin x = 1$

$\sin x - \sin^2 x = 0$

$\sin x(1 - \sin x) = 0$

$\sin x = 0$ $\quad$ $\sin x = 1$

$\sin x = 0$ $\quad$ $\boxed{x = 0, \pi}$

$\sin x = 1$ $\quad$ $\boxed{x = \frac{\pi}{2}}$

11. $\tan^2 x - 1 = 0$

$\tan^2 x = 1$

$\tan x = \pm 1$

$\boxed{x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$

12. $3\sec^2 x - 2\tan^2 x - 4 = 0$

$3(1+\tan^2 x) - 2\tan^2 x - 4 = 0$

$3 + 3\tan^2 x - 2\tan^2 x - 4 = 0$

$\tan^2 x - 1 = 0$

$\tan^2 x = 1$

$\tan x = \pm 1$ $\quad$ $\boxed{x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$

13. $2\cos^2 x - \cos x = 1$

$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

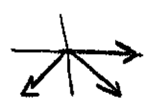
$2\cos x + 1 = 0$ $\quad$ $\cos x - 1 = 0$

$\cos x = -\frac{1}{2}$ $\quad$ $\cos x = 1$

$\boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3}}$ $\quad$ $\boxed{x = 0}$

Solve for all real numbers.

14. $\sqrt{2}\sin x + 1 = 0$



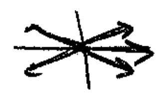
$\sqrt{2}\sin x = -1$

$\sin x = \frac{-1}{\sqrt{2}}$

$\sin x = \frac{-\sqrt{2}}{2}$

$\boxed{x = \frac{5\pi}{4} + 2\pi n, \quad x = \frac{7\pi}{4} + 2\pi n}$

15. $4\cos^2 x - 3 = 0$



$4\cos^2 x = 3$

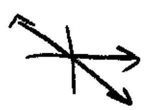
$\cos^2 x = \frac{3}{4}$

* every quadrant

$\cos x = \pm\frac{\sqrt{3}}{2}$

$\boxed{x = \frac{\pi}{6} + \pi n, \quad x = \frac{5\pi}{6} + \pi n}$

16. $\cot x + 1 = 0$



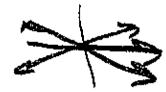
$\cot x = -1$

$\tan x = -1$

$\boxed{x = \frac{3\pi}{4} + \pi n}$

17. $4\tan^2 x - 1 = \tan^2 x$

$3\tan^2 x - 1 = 0$



$3\tan^2 x = 1$

$\tan^2 x = \frac{1}{3}$

* every quadrant

$\tan x = \pm\frac{\sqrt{3}}{3}$

$\boxed{x = \frac{\pi}{6} + \pi n, \quad x = \frac{5\pi}{6} + \pi n}$

18. $\sin\left(\frac{x}{2}\right) = 0$ $\quad$ let $u = \frac{x}{2}$

$\sin u = 0$

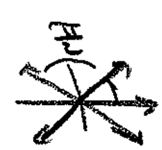
$u = 0 + \pi n$

$\frac{x}{2} = 0 + \pi n$

$\boxed{x = 2\pi n}$

19. $2\sin^2 3x - 1 = 0$

let $u = 3x$



$2\sin^2 u - 1 = 0$

$\sin^2 u = \frac{1}{2}$

$\sin u = \pm\frac{\sqrt{2}}{2}$

$u = \frac{\pi}{4} + \frac{\pi}{2}n$

$3x = \frac{\pi}{4} + \frac{\pi}{2}n$ $\quad$ $\boxed{x = \frac{\pi}{12} + \frac{\pi}{6}n}$