Synthetic Division

Synthetic division is a shortcut for polynomial division.

It only works for linear binomials. i.e. (x + 2) (2x – 3)

Divide: $(2x^2 + 7x + 9) \div (x + 2)$

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$$\begin{array}{r|rrr} -2 & 2 & 7 & 9 \\ \hline & & & \end{array}$$

Write the coefficients of the dividend (including 0 placeholders if needed)

Use the opposite sign of the number in the divisor.

$$\begin{array}{r|rrr} -2 & 2 & 7 & 9 \\ \hline & 2 & & \end{array}$$

Step 1: Bring down the first coefficient

$$
\begin{array}{r|rrr}
-2 & 2 & 7 & 9 \\
 & & -4 & \\
\hline
 & 2 & &
\end{array}
$$

Step 2: Multiply the divisor with the coefficient. Place it in the next column.
-2 x 2 = -4

$$
\begin{array}{r|rrr}
-2 & 2 & 7 & 9 \\
 & & -4 & \\
\hline
 & 2 & 3 &
\end{array}
$$

Step 3: Add 7 + (-4) = 3

$$\begin{array}{r|rrr} -2 & 2 & 7 & 9 \\ & & -4 & -6 \\ \hline & 2 & 3 & \end{array}$$

Step 4: Multiply the divisor with the new coefficient. Place it in the next column.

-2 x 3 = -6

$$\begin{array}{r|rr|r} -2 & 2 & 7 & 9 \\ & & -4 & -6 \\ \hline & 2 & 3 & 3 \end{array}$$

Step 5: Add 9 + (-6) = 3

This last number is the remainder.

$$
\begin{array}{r|rrr}
-2 & 2 & 7 & 9 \\
 & & -4 & -6 \\
\hline
 & 2 & 3 & \boxed{3}
\end{array}
$$

Degree 1

Degree 0

Remainder

Final answer: put a variable with correct degree on each coefficient.

Solution: $2x + 3 + \frac{3}{x+2}$

Divide: $(3x^4 - x^3 + 5x - 1) \div (x - 3)$

$$\begin{array}{c|ccccc} 3 & 3 & -1 & 0 & 5 & -1 \\ & & & & & \\ \hline & & & & & \end{array}$$

Write the coefficients of the dividend (including 0 placeholders if needed)

Use the opposite sign of the number in the divisor.

$$\begin{array}{c|ccccc} 3 & 3 & -1 & 0 & 5 & -1 \\ & & & & & \\ \hline & 3 & & & & \end{array}$$

Step 1: Bring down the first coefficient

$$
\begin{array}{r|rrrrr}
\color{red}{3} & 3 & -1 & 0 & 5 & -1 \\
 & & 9 & & & \\
\hline
 & 3 & & & &
\end{array}
$$

Step 2: Multiply the divisor with the coefficient. Place it in the next column.
3 x 3 = 9

$$
\begin{array}{r|rrrrr}
\color{red}{3} & 3 & -1 & 0 & 5 & -1 \\
 & & 9 & & & \\
\hline
 & 3 & 8 & & &
\end{array}
$$

Step 3: Add -1 + 9 = 8

3	3	-1	0	5	-1
		9	24	72	231
	3	8	24	77	230

Repeat steps 2 and 3 as necessary

3	3	-1	0	5	-1
		9	24	72	231
	3	8	24	77	230
	Degree 3	Degree 2	Degree 1	Degree 0	Remainder

Write final answer with correct degrees.

Solution: $3x^3 + 8x^2 + 24x + 77 + \frac{230}{x-3}$

Divide: $(3x^2 + 9x - 2) \div (3x - 1)$

The divisor must have a leading coefficient of 1. Divide it by 3.

Divide: $(3x^2 + 9x - 2) \div (x - 1/3)$

$$\begin{array}{r|rrr} 1/3 & 3 & 9 & -2 \\ & & 1 & 3\frac{1}{3} \\ \hline & 3 & 10 & \boxed{1\frac{1}{3}} \end{array}$$

Use synthetic division like normal.

Solution: $3x + 10 + \dfrac{1\frac{1}{3}}{x - \frac{1}{3}}$

Note: the remainder is over the new divisor.

Factor Theorem: A binomial is a factor of a polynomial (like $(x-2)(x-4)$ are factors of $x^2 - 6x + 8$) if synthetic division has a remainder of zero.

Is $(x-2)$ a factor of $3x^3 + 2x^2 - 33$?

$$
\begin{array}{r|rrrr}
2 & 3 & 2 & 0 & -33 \\
 & & 6 & 16 & 32 \\
\hline
 & 3 & 8 & 16 & \boxed{-1}
\end{array}
$$

$(x-2)$ is **not** a factor of the polynomial.

Is x = -4 a solution to the polynomial
$P(x) = x^3 + 2x^2 – 3x + 20$?

If x = -4 is a solution, then (x + 4) must be a factor.
Use the factor theorem.

$$
\begin{array}{r|rrrr}
-4 & 1 & 2 & -3 & 20 \\
 & & -4 & 8 & -20 \\
\hline
 & 1 & -2 & 5 & \boxed{0}
\end{array}
$$

x = -4 is a solution to P(x).

Remainder Theorem: Synthetic division can evaluate a function's value (i.e. f(3)). The answer is the remainder.

Note: you do not use the opposite sign of the #

If $P(x) = 3x^4 - 25x^2 + 4$, find $P(-3)$

$$\begin{array}{r|rrrrr} -3 & 3 & 0 & -25 & 0 & 4 \\ & & -9 & 27 & -6 & 18 \\ \hline & 3 & -9 & 2 & -6 & \boxed{22} \end{array}$$

$P(-3) = 22$