

U4 Functions Review

$f(x) = x^3$ $g(x) = x+1$ $j(x) = \sqrt{x}$ $p(x) = 3x^2$

1. $(p-g)(-1)$
 $(p-g)(x) = 3x^2 - (x+1)$
 $= 3x^2 - x - 1$
 $(p-g)(-1) = 3(-1)^2 - (-1) - 1$
 $3 + 1 - 1 = 3$

8. $f(x+h) = (x+h)^3$
 $(x+h)(x+h)(x+h)$
 $(x+h)(x^2 + 2xh + h^2)$
 $x^3 + 2x^2h + xh^2 + x^2h + 2xh^2 + h^3$
 $x^3 + 3x^2h + 3xh^2 + h^3$

2. $(jg)(a) = \sqrt{a}(a+1)$
 or $a^{1/2}(a+1)$
 $a^{3/2} + a^{1/2}$

9. $p(x+h) - p(x)$
 $3(x+h)^2 - 3x^2$
 $3(x^2 + 2xh + h^2) - 3x^2$
 $3x^2 + 6xh + 3h^2 - 3x^2$
 $6xh + 3h^2$

3. $f(p(-2))$
 $p(-2) = 3(-2)^2 = 12$
 $f(12) = 12^3 = 1728$

10. $k(x) = \sqrt{x^2+1}$
 $k(x) = f(g(h(x)))$

4. $j(p(g(3)))$
 $g(3) = 3+1 = 4$
 $p(4) = 3(4)^2 = 48$
 $j(48) = \sqrt{48} = 4\sqrt{3}$

$f(x) = \sqrt{x}$
 $g(x) = x+1$
 $h(x) = x^2$

5. $p(j(x)) = 3(\sqrt{x})^2 = 3x$

11. $f(x) = 2x^2 - x - 7$
 $f(x+h) - f(x)$
 h

6. $(\frac{f}{g})(x) = \frac{x^3}{x+1}$

$f(x+h) = 2(x+h)^2 - (x+h) - 7$
 $= 2(x^2 + 2xh + h^2) - x - h - 7$
 $= 2x^2 + 4xh + 2h^2 - x - h - 7$

7. $p(j(g(x)))$
 $j(g(x)) = \sqrt{x+1}$
 $p(\sqrt{x+1}) = 3(\sqrt{x+1})^2$
 $= 3(x+1) = 3x+3$

$f(x+h) - f(x) = (2x^2 + 4xh + 2h^2 - x - h - 7) - (2x^2 - x - 7)$
 h h
 $= 4xh + 2h^2 - h = h(4x + 2h - 1)$
 h h
 $4x + 2h - 1$

12. see end of key

13. $f(x) = \begin{cases} (x+2)^2 - 2 & \text{if } x < 0 \\ \frac{2}{5}x + 3 & \text{if } 0 \leq x < 5 \\ 3 & \text{if } 5 \leq x < 8 \end{cases}$

15. A. $y = \sqrt[3]{x-5}$
 parent: $y = \sqrt[3]{x}$
 transformation: right 5
 domain: $(-\infty, \infty)$
 range: $(-\infty, \infty)$

14. A. $f(x) = \frac{x+3}{3}$ $g(x) = 3x-3$

B. $y = \frac{1}{3}e^{x+2}$
 parent: $y = e^x$
 transformations: left 2,
 vertical compression of $\frac{1}{3}$
 domain: $(-\infty, \infty)$
 range: $(0, \infty)$

$f(g(x)) = \frac{(3x-3)+3}{3} = \frac{3x}{3} = x$

$g(f(x)) = 3\left[\frac{x+3}{3}\right] - 3 = x+3-3 = x$

yes! $f(g(x)) = g(f(x)) = x$

C. $f(x) = \frac{1}{(x+3)} - 5$

B. $f(x) = \sqrt{x+2} + 3$ $g(x) = x^2 - 6x + 7$
 $f(g(x)) = \sqrt{x^2 - 6x + 7} + 2 + 3$
 $= \sqrt{x^2 - 6x + 9} + 3$
 $= \sqrt{(x-3)^2} + 3$
 $= |x-3| + 3 = x$

parent: $y = x^2$
 transformations:
 left 3, down 5
 domain: $(-\infty, -3) \cup (-3, \infty)$
 range: $(-\infty, -5) \cup (-5, \infty)$

$g(f(x)) = (\sqrt{x+2} + 3)^2 - 6(\sqrt{x+2} + 3) + 7$

$(\sqrt{x+2} + 3)(\sqrt{x+2} + 3) - 6\sqrt{x+2} - 18 + 7$
 $(\sqrt{x+2})^2 + 3\sqrt{x+2} + 3\sqrt{x+2} + 9 - 6\sqrt{x+2} - 18 + 7$
 $x+2 + 6\sqrt{x+2} + 9 - 6\sqrt{x+2} - 18 + 7$
 $x+2+9-18+7$

D. $f(x) = 2\sin(-x) + 3$

parent: $y = \sin x$
 transformations:
 vertical stretch of 2,
 reflection about y-axis,
 up 3

yes $f(g(x)) = g(f(x)) = x$

domain: $(-\infty, \infty)$
 range: $[-1, 5]$

E. $f(x) = \frac{1}{(x-4)^2} - 1$

parent: $y = \frac{1}{x^2}$

transformations: right 4,
down 1

domain: $(-\infty, 4) \cup (4, \infty)$

range: $(-1, \infty)$

19. A. $f(5) = 0$

B. $f(x) = -1$ when

$x \approx 0.6, -2.8, 4$

C. $(-\infty, -1.5) \cup (2, 4)$

D. $(-1.5 \cup 2) \cup (4, \infty)$

E. $(-\infty, \infty)$

F. $x = -1.5$

G. local mins: $-1, -4$

local max(s): 0

H. $f(-2) < 0$ negative

I. $f(x) < 0$ on $(-3, 2)$

F. $y = -x^3 - 2$

parent $y = x^3$

transformations:

reflection about x-axis

down 2

domain: $(-\infty, \infty)$

range: $(-\infty, \infty)$

16. $y = \sqrt{x}$ square root

$y = \tan x$ tangent

$y = \ln x$ natural log

$y = \frac{1}{x}$ reciprocal

17. $y = e^x$ exponential

$y = \ln x$ natural log

$y = \sqrt{x}$ square root

$y = [x]$ greatest integer

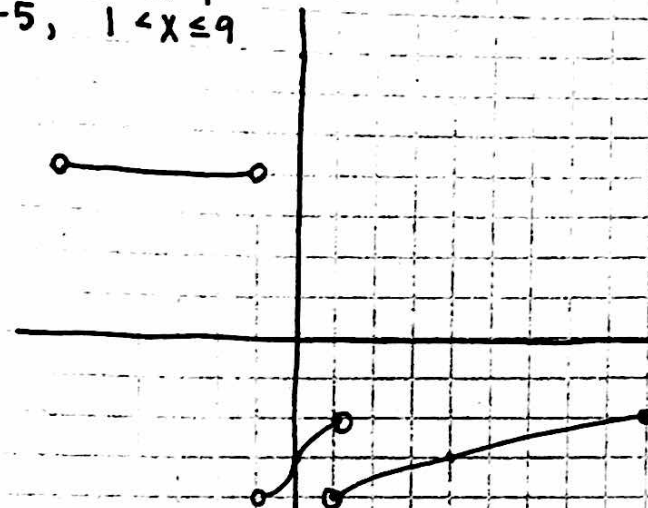
18. $y = \tan x$ tangent

$y = [x]$ greatest integer

$y = \frac{1}{x}$ reciprocal

$y = x^2$ volcano

$$f(x) = \begin{cases} 4 & -6 < x < -1 \\ x^2 - 3 & -1 < x < 1 \\ \sqrt{x} - 5 & 1 < x \leq 9 \end{cases}$$



A. Domain: $(-6, -1) \cup (-1, 1) \cup (1, 9]$

B. Range: $(-4, -2] \cup [4]$

C. $f(\frac{1}{2}) = (\frac{1}{2})^2 - 3$

$\frac{1}{4} - 3 = \frac{-12}{4} = \frac{-23}{4}$