<u>Graphing Rational Functions</u>

Rational Function: an equation of the form $f(x) = \frac{p(x)}{q(x)}$ where p(x) and q(x) are polynomial functions and $q(x) \neq 0$.

Examples:

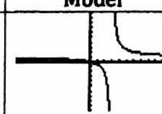
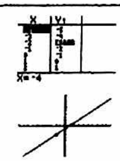
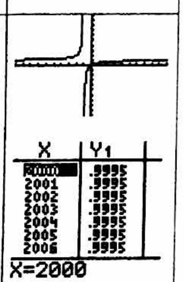
$f(x) = \frac{x}{x+3}$ $\quad g(x) = \frac{5}{x-6}$ $\quad h(x) = \frac{x+4}{(x-1)(x+4)}$

No denominator can be zero. In the examples above, the functions are not defined at

$$x = -3, x = 6, \text{and } x = 1 \text{ and } x = -4.$$

The graphs of rational functions may have breaks in ***continuity***. *(Not traceable)*

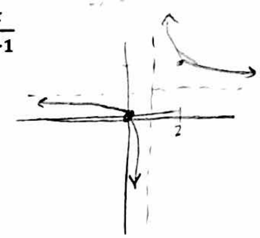
Breaks in continuity can appear as ***asymptotes*** or as ***points of discontinuity***.

Property	Words	Example	Model
Vertical Asymptote	If the rational expression of a function is written in simplest form and the function is undefined for $x = a$, then $x = a$ is a vertical symptote.	For $f(x) = \frac{x}{x-3}$ $x = 3$ is an asymptote.	
Point of Discontinuity (Hole)	If the original function is undefined for $x = a$ but the rational expression of the function in simplest form is defined for $x = a$, then there is a hole in the graph at $x = a$.	For $f(x) = \frac{(x+2)(x-1)}{x+2}$ Can be simplified to $f(x) = x - 1$ So, $x = -2$ represents a hole in the graph. Find the exact location by plugging in -2 to the simplified function.	
Horizontal Asymptote	For $f(x) = \frac{ax^n + \cdots}{bx^m + \cdots}$ Given n and m where n is the highest power of x in the numerator and m is the highest power of x in the denominator. •If $n < m$, then horizontal symptote is $y = 0$. •If $n = m$, then horizontal asymptote is $y = \frac{a}{b}$. •If $n > m$, then there is no horizontal asymptote.	For $f(x) = \frac{x}{x+1}$, $\frac{x}{x}$ $n = m$ $y = \frac{1}{1} = 1$ Horizontal Asymptote at $y = 1$	

Determine the equations of any vertical asymptotes, horizontal asymptotes and the values of x for any holes in the following graphs, then graph!!

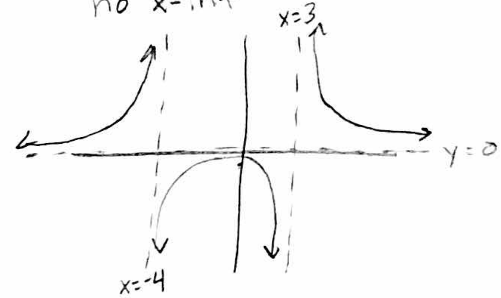
1. $h(x) = \frac{x}{x-1}$

VA $x=1$
HA: $y=1$
x-int: $x=0$



2. $f(x) = \frac{6}{(x-3)(x+4)}$

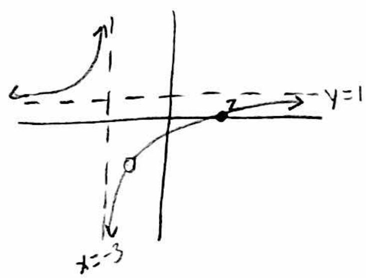
VA: $x=3,-4$
HA: $y=0$
no holes
no x-int



3. $f(x) = \frac{x^2-4}{x^2+5x+6}$

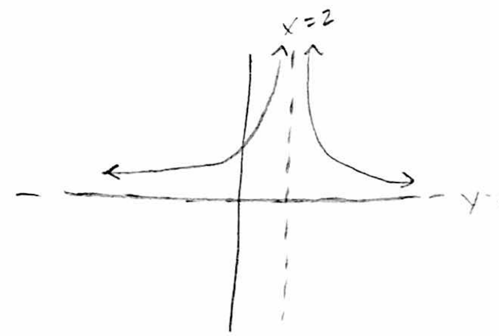
$f(x) = \frac{(x-2)(x+2)}{(x+3)(x+2)} = \frac{(x-2)}{(x+3)}$

VA: $x=-3$ $\quad$ x-int: $x=2$
HA: $y=1$
Hole: $(-2,-4)$



4. $f(x) = \frac{2}{(x-2)^2}$

VA: $x=2$
HA: $y=0$



Slant Asymptote:

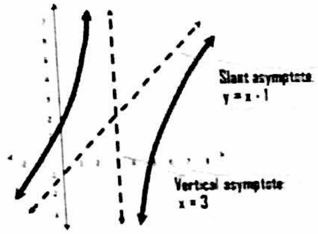
For $f(x) = \frac{ax^n+\cdots}{bx^m+\cdots}$

If n is 1 more than m (m+1=n) then there is a slant (oblique) asymptote.

Find this asymptote by dividing the numerator of the expression by the denominator (use synthetic division). Drop the remainder and what is left is the equation of the asymptote.

Ex: $f(x) = \frac{x^2-4x-5}{x-3}$

- Degree on top is 1 more than degree on bottom.
- Divide numerator by denominator
- Results is x-1 (dropped the remainder).
- → y=x-1 is the equation of the asymptote.



Ex: $f(x) = \frac{3x^2+x-2}{2x+6} = \frac{(3x-2)(x+1)}{2(x+3)}$ x-int: $x = \frac{2}{3}, -1$

VA: $2x+6=0$

$2x=-6$

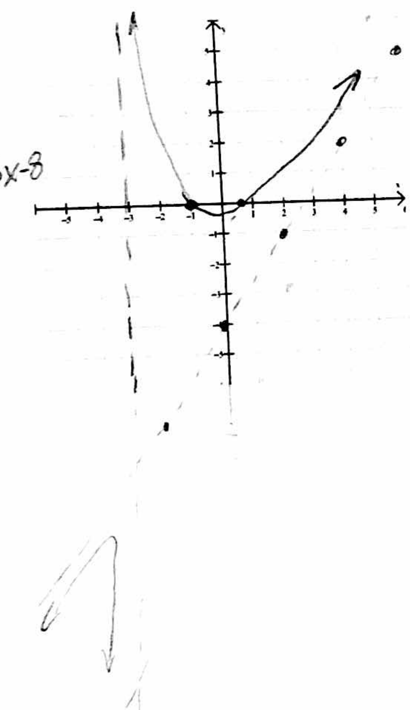
$x = -\frac{6}{2} = -3$

Slant: $y=3x-8$

$$\begin{array}{r|rrr} -3 & 3 & 1 & -2 \\ & \downarrow & -9 & 24 \\ \hline & 3 & -8 & \boxed{22} \end{array}$$

$3x-8$ (drop remainder)

$\boxed{y = \frac{3}{2}x - 4}$

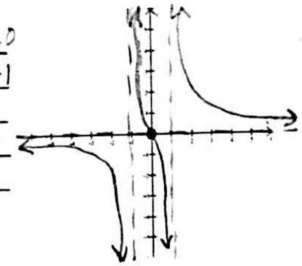


More Practice:

Find: (a) the x-intercepts, y-intercepts, (b) the vertical asymptote(s), (c) the horizontal asymptote or slant asymptote, (d) the holes, (e) any additional points needed, (f) graph the function.

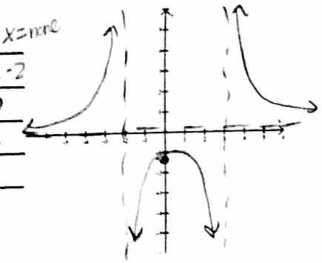
1. $f(x) = \frac{2x}{x^2-1} = \frac{2x}{(x-1)(x+1)}$

(a) y=0, x=0
(b) x=1,-1
(c) y=0
(d) none
(e)



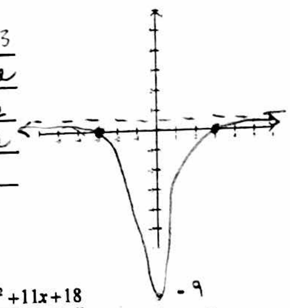
2. $y = \frac{8}{x^2-x-6} = \frac{8}{(x-3)(x+2)}$

(a) $y=\frac{-4}{3}$, x=none
(b) x=3,-2
(c) y=0
(d) none
(e)



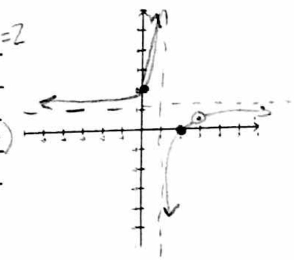
3. $f(x) = \frac{x^2-9}{2x^2+1} = \frac{(x-3)(x+3)}{2x^2+1}$

(a) x=3,-3
(b) none
(c) y=1/2
(d) none
(e)



4. $p(x) = \frac{x^2-5x+6}{x^2-4x+3} = \frac{(x-2)(x-3)}{(x-3)(x-1)} = \frac{x-2}{x-1}$

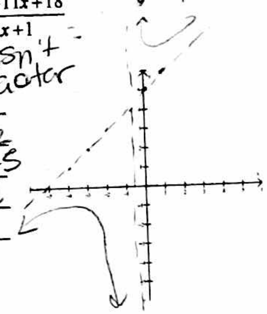
(a) x=2, y=2
(b) x=1
(c) y=1
(d) (3, 1/2)
(e)



5. $g(x) = \frac{2x^2+11x+18}{2x+1}$

quadratic eq doesn't factor

(a) none
(b) x=-1/2
(c) y=x+5
(d) none
(e)



$$\begin{array}{r|rrr} -\frac{1}{2} & 2 & 11 & 18 \\ & \downarrow & -1 & -5 \\ \hline & 2 & 10 & \boxed{13} \end{array}$$

$2x+10$

$x+5$

6. $y = \frac{x-4}{x^2-3x} = \frac{x-4}{x(x-3)}$

(a) 4
(b) x=0,3
(c) y=0
(d) none
(e)