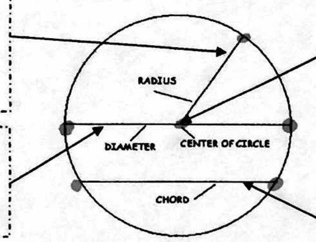
6.8 Chords & Arcs of Circles

SWBAT solve for unknown variables using theorems about chords and arcs of circles.

Any segment with endpoints that are the center and a point on the circle is a radius

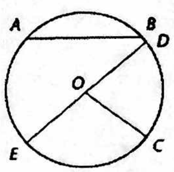
A segment that passes through the center is a diameter of a circle.



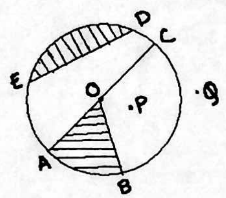
The given point is called the center. This point names the circle.

Any segment with endpoints that are on a circle is called a chord.

Example 1: Name the circle, a radius, a chord, and a diameter of the circle.



Circle: ⊙O
Radius: $\overline{OC}$ $\overline{OD}$, $\overline{OE}$
Chord: $\overline{AB}$
Diameter: $\overline{ED}$



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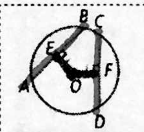
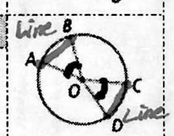
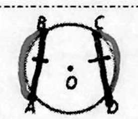
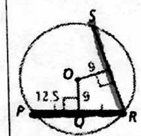
Circle: ⊙O
Radius: $\overline{AO}$, $\overline{OB}$, $\overline{OC}$
Chord: $\overline{ED}$
Diameter: $\overline{AC}$

Since a diameter is composed of two radii, then **d = 2r** and **r = d/2**

Theorem 1:	Converse Theorem 1:	
Within a circle or in congruent circles, chords equidistant from the center or centers are congruent. If $OE \cong OF$, then $\overline{AB} \cong \overline{CD}$.	Within a circle or in congruent circles, congruent chords are equidistant from the center (or centers). If $\overline{AB} \cong \overline{CD}$, then $OE \cong OF$.	
Theorem 2:	**Converse Theorem 2:**	
Within a circle or in congruent circles, congruent central angles have congruent arcs. If $\angle AOB \cong \angle COD$, then $\overparen{AB} \cong \overparen{CD}$.	Within a circle or in congruent circles, congruent arcs have congruent central angles. If $\overparen{AB} \cong \overparen{CD}$, then $\angle AOB \cong \angle COD$.	
Theorem 3:	**Converse Theorem 3:**	
Within a circle or in congruent circles, congruent central angles have congruent chords. If $\angle AOB \cong \angle COD$, then $\overline{AB} \cong \overline{CD}$.	Within a circle or in congruent circles, congruent chords have congruent central angles. If $\overline{AB} \cong \overline{CD}$, then $\angle AOB \cong \angle COD$.	 Line Line
Theorem 4:	**Converse Theorem 4:**	
Within a circle or in congruent circles, congruent chords have congruent arcs. If $\overline{AB} \cong \overline{CD}$, then $\overparen{AB} \cong \overparen{CD}$.	Within a circle or in congruent circles, congruent arcs have congruent chords. If $\overparen{AB} \cong \overparen{CD}$, then $\overline{AB} \cong \overline{CD}$.	

Example 2: The following chords are equidistant from the center of the circle.

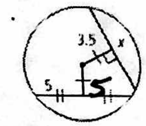
a) What is the length of RS?



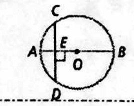
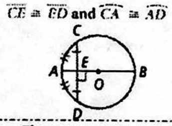
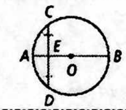
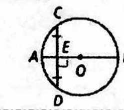
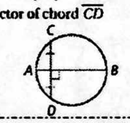
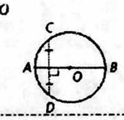
PR = 12.5 + 12.5
= 25

SR = 25 b/c of Thm. 1

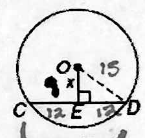
b) Solve for x.



X = 5 + 5
= 10

Theorem 5:	If . . .	Then . . .
In a circle, if a diameter is perpendicular to a chord, then it bisects the chord and its arc.	$\overline{AB}$ is a diameter and $\overline{AB} \perp \overline{CD}$ 	$\overline{CE} \cong \overline{ED}$ and $\overparen{CA} \cong \overparen{AD}$ 
Theorem 6:	**If . . .**	**Then . . .**
In a circle, if a diameter bisects a chord that is not a diameter, then it is perpendicular to the chord.	$\overline{AB}$ is a diameter and $\overline{CE} \cong \overline{ED}$ 	$\overline{AB} \perp \overline{CD}$ 
Theorem 7:	**If . . .**	**Then . . .**
In a circle, the perpendicular bisector of a chord contains the center of the circle.	$\overline{AB}$ is the perpendicular bisector of chord $\overline{CD}$ 	$\overline{AB}$ contains the center of ⊙O 

Example 3: In ⊙O, $\overline{CD} \perp \overline{OE}$, $OD = 15$, and $CD = 24$. Find x.



$\underbrace{\quad}_{24}$

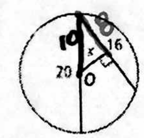
$x^2 + 12^2 = 15^2$

$x^2 + 144 = 225$

$x^2 = 81$

$\boxed{x = 9}$

Example 4: Find the value of x to the nearest tenth.



$x^2 + 8^2 = 10^2$

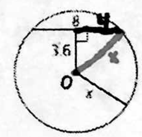
$x^2 + 64 = 100$

$x^2 = 36$

$\boxed{x = 6}$

You Try! Find the value of x to the nearest tenth.

a)



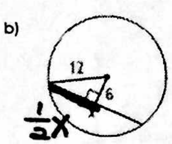
$(3.6)^2 + 4^2 = x^2$

$\boxed{x = 5.381}$

Jose can't round!

$\boxed{5.4}$

b)



$\frac{1}{2}x$

$(\frac{1}{2}x)^2 + 6^2 = 12^2$

$\frac{1}{4}x^2 + 36 = 144$

$\frac{1}{4}x^2 = 108$

$x^2 = 432$

$x = \sqrt{432} = \boxed{20.8}$