Math 3: Unit 7 Circle Review Worksheet   Name _____

Equation of a Circle: $(x - h)^2 + (y - k)^2 = r^2$, Center = (h, k) and Radius = r

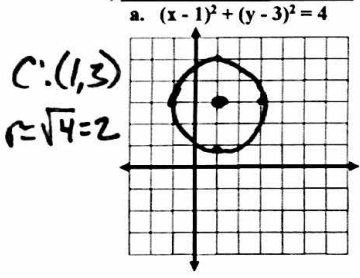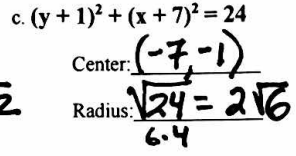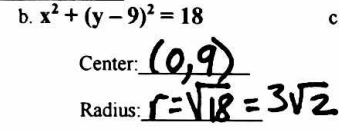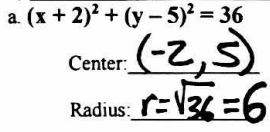1) Graph the following circle:

a. $(x - 1)^2 + (y - 3)^2 = 4$

C: (1,3)
$r = \sqrt{4} = 2$

b. $(x - 4)^2 + (y - 2)^2 = 9$

C: (4,2)
r = 3

c. $(y + 3)^2 + (x + 1)^2 = 16$

C: (-1,-3)
$r = \sqrt{16} = 4$
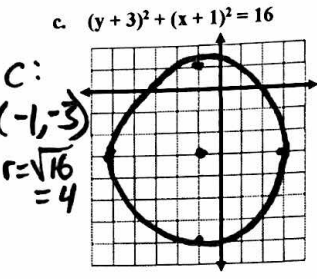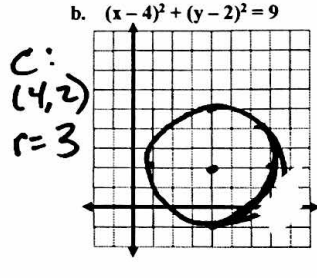
2) For each circle: Identify its center and radius.

a. $(x + 2)^2 + (y - 5)^2 = 36$

Center: (-2, 5)

Radius: $r = \sqrt{36} = 6$

b. $x^2 + (y - 9)^2 = 18$

Center: (0, 9)

Radius: $r = \sqrt{18} = 3\sqrt{2}$

c. $(y + 1)^2 + (x + 7)^2 = 24$

Center: (-7, -1)

Radius: $\sqrt{24} = 2\sqrt{6}$
6.4

3) Write the equation of the following circles:

C: (-5, 4)
r = 5

$(x + 5)^2 + (y - 4)^2 = 25$

C: (1, -3)
r = 4

$(x - 1)^2 + (y + 3)^2 = 16$
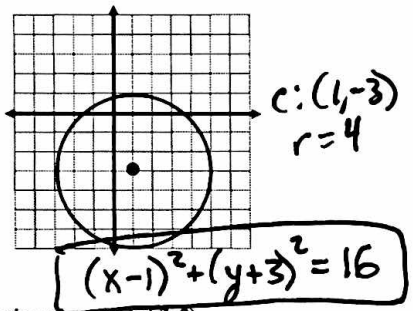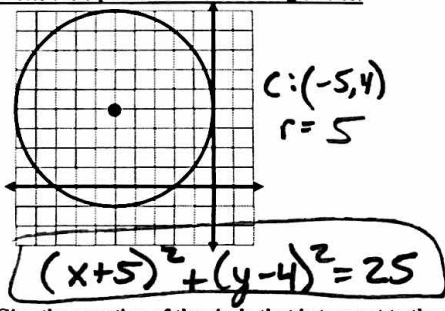
4) Give the equation of the circle that is tangent to the y-axis and center is (-3, 2).

5) Give the equation of the circle that is tangent to the x-axis and center is (5, -7).

Finding Circles in Standard Form: COMPLETE THE SQUARE on the x terms and y terms separately.

EXP: $x^2 + y^2 + 6x - 8y - 11 = 0$

$(x^2 + 6x) + (y^2 - 8y) = 11$ x-terms: $6 \div 2 = 3$ and $(3)^2 = 9$ y-terms: $-8 \div 2 = -4$ and $(-4)^2 = 16$

$(x^2 + 6x + 9) + (y^2 - 8y + 16) = 11 + 9 + 16$ Factor

$(x + 3)^2 + (y - 4)^2 = 36$ Center: (-3, 4) Radius: 6

6) Find the standard form, center, and radius of the following circles:

6a) $x^2 + y^2 - 4x + 10y - 7 = 0$

6b) $x^2 + 8x + y^2 + 5y - 2 = 0$

$x^2 + 8x + 16 + y^2 + 5y + 6.25 = 2 + 16 + 6.25$

$(x + 4)^2 + (y + \frac{5}{2})^2 = 24.25$

as a fraction =

Center: $(-4, -\frac{5}{2})$ Radius: $\frac{\sqrt{97}}{\sqrt{4}} = \frac{\sqrt{97}}{2}$ $\frac{97}{4}$

6c) $x^2 - 2x + y^2 + 12y + 18 = 0$

6d) $x^2 - 10x + y^2 - 6y + 9 = 0$

Center:_____ Radius:_____

Center:_____ Radius:_____

7) Give the equation of the circle whose

a. Center is (4,-2) and goes through (2, 5)

$(x - h)^2 + (y - k)^2 = r^2$

$(x - 4)^2 + (y + 2)^2 = 53$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

b. Center is (3, 3) and goes through (1, 1)

9) Give the equation of a circle whose

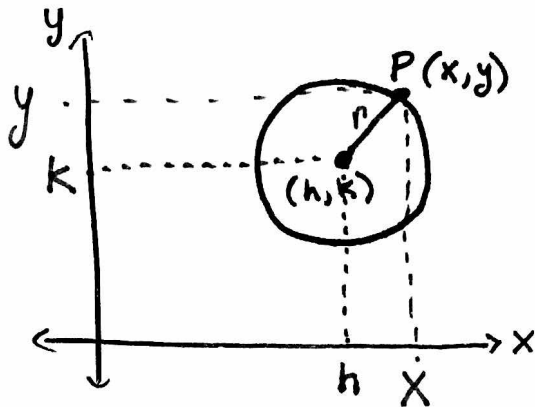a. Endpoints of a diameter at (-4, 1) and (4, -5)

b. Endpoints of a diameter at (7, -2) and (3, -8)

Equations of Circles

Standard Form of Circles:

$$(x-h)^2 + (y-k)^2 = r^2$$

center:	radius:	Point on circle:
(h,k)	r	(x,y)

Ex: Write eqn. of circle w/ center $(0,0)$ and radius $= 8$.
$\underset{(h,k)}{} \qquad \underset{r}{}$

$$(x-0)^2 + (y-0)^2 = 8^2$$

$$\boxed{x^2 + y^2 = 64}$$

Ex: Write eqn of circle w/ Center $(6,-3)$ w/ radius $= 10$.
$\underset{(h,k)}{} \qquad \underset{r}{}$

$$\boxed{(x-6)^2 + (y+3)^2 = 100}$$

Ex Find the center & radius.

$$x^2 + y^2 = \frac{9}{4}$$

$$\boxed{\begin{array}{l} \text{center: } (0,0) \\ \text{radius: } 3/2 \end{array}}$$

$\sqrt{r^2} = \sqrt{\frac{9}{4}} = \frac{\sqrt{9}}{\sqrt{4}} = \frac{3}{2}$

Graph:

Ex: Find center & radius.

$$(x+7)^2 + (y-5)^2 = 12$$

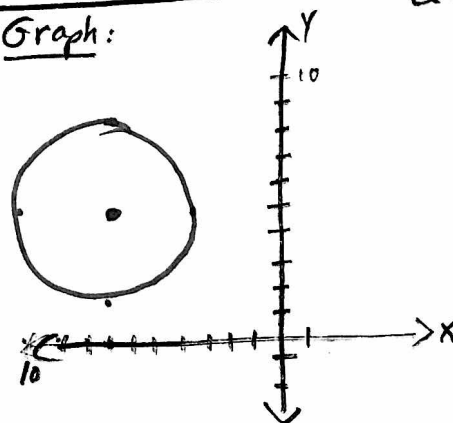$$\boxed{\begin{array}{l} \text{center: } (-7,5) \\ \text{radius: } 2\sqrt{3} \end{array}}$$

$\sqrt{r^2} = \sqrt{12}$
$= \sqrt{4 \cdot 3}$
$= 2\sqrt{3}$

Graph:

Write equation of a circle

Ex: Write eqn of circle w/ center $(8, -7)$ and goes through the point $(-3, 1)$.

$(8, -7)$ is (h, k) $(-3, 1)$ is (x, y)

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(-3-8)^2 + (1 - ^-7)^2 = r^2$$
$$(-11)^2 + (8)^2 = r^2$$
$$121 + 64 = r^2$$
$$\boxed{185 = r^2}$$

Final eqn:
$$\boxed{(x-8)^2 + (y+7)^2 = 185}$$

Ex: Write eqn of circle w/ center $(0, 4)$ and goes through point $(-2, -5)$.

h k x y

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(0 - ^-2)^2 + (4 - ^-5)^2 = r^2$$
$$(2)^2 + (9)^2 = r^2$$
$$85 = r^2$$
$$\boxed{x^2 + (y-4)^2 = 85}$$

Ex: Write eqn of circle w/ endpoints of diameter at $(-6, 5)$ and $(4, -3)$. (x, y)

Need center first! Find midpt:

$$\left(\frac{-6+4}{2}, \frac{5+^-3}{2}\right) = \left(\frac{-2}{2}, \frac{2}{2}\right) = \underset{(h, k)}{(-1, 1)}$$

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(4 - ^-1)^2 + (^-3-1)^2 = r^2$$
$$(5)^2 + (-4)^2 = r^2$$
$$25 + 16 = \boxed{41} = r^2$$

Final eq:
$$\boxed{(x+1)^2 + (y-1)^2 = 41}$$

write eqn in std. form

Ex: Write eqn in standard form. Then give center + radius.

a) $x^2 + y^2 + 4x - 8y + 16 = 0$.

$(x^2 + 4x + 4) + (y^2 - 8y + 16) = -16 + 4 + 16$

$\frac{+4}{2} = (2)^2 = 4$ $\qquad \frac{-8}{2} = (-4)^2 = 16$

$(x + 2)^2 + (y - 4)^2 = 4$

center: $(-2, 4)$ $\qquad$ radius: $\sqrt{4} = 2$

b) $x^2 + y^2 - 6x - 2y + 4 = 0$

$(x^2 - 6x + 9) + (y^2 - 2y + 1) = -4 + 9 + 1$

$(x - 3)^2 + (y - 1)^2 = 6$

center: $(3, 1)$ $\qquad$ radius: $\sqrt{6}$

Writing an Equation with Two Points on the Circle	**Midpoint Formula**
Find the midpoint (radius) between the two endpoints, and then follow steps 1-4.	$\left(\dfrac{X_1+X_2}{2}, \dfrac{y_1+y_2}{2}\right)$ This will be the center.

Example 5: Write the equation of a circle with endpoints of diameter at (-6, 5) and (4, -3).

Center: $\left(\dfrac{-6+4}{2}, \dfrac{5+^-3}{2}\right)$

$(-1, 1)$
(h, K)

Eqn: $(X-h)^2+(y-K)^2=r^2$

(X, y)

$(4+1)^2+(-3-1)^2=r^2$

$(5)^2+(-4)^2=r^2$

$41 = r^2$

$(x+1)^2+(y-1)^2=41$

Writing the Equation of a Circle in Standard Form

Step 1:	Group x's and group y's together.
Step 2:	Move any constants to the right side of the equation.
Step 3:	Use complete the square to make a perfect square trinomial for the x's and then again for the y's. *Remember, whatever you do to one side of the equation, you must do to the other!*
Step 4:	Simplify factors into standard form of a circle!

Example 5: Write the equation of a circle in standard form. Then, state the center and the radius.

a) $x^2 + y^2 + 4x - 8y + 16 = 0$

$(x^2+4x+4)+(y^2-8y+16) = ^-16+4+16$

$\dfrac{4}{2}=(2)^2=4$ $\dfrac{-8}{2}=(-4)^2=16$

$(x+2)^2+(y-4)^2=4$ eqn.

center: $(-2, 4)$
radius: 2

b) $x^2 + y^2 + 6x - 4y = 0$

$(x+3)^2+(y-2)^2=13$

$c: (-3, 2)$
$r: \sqrt{13}$

c) $x^2 + y^2 - 6x - 2y + 4 = 0$

$(x^2-6x+9)+(y^2-2y+1)=^-4+9+1$

$\dfrac{-6}{2}=(-3)^2=9$ $\dfrac{-2}{2}=(-1)^2=1$

$(x-3)^2+(y-1)^2=6$

center: $(3, 1)$
radius: $\sqrt{6}$

d) $x^2 + y^2 + 8x - 10y - 4 = 0$

$(x+4)^2+(y-5)^2=45$

$c: (-4, 5)$
$r: \sqrt{45}=3\sqrt{5}$

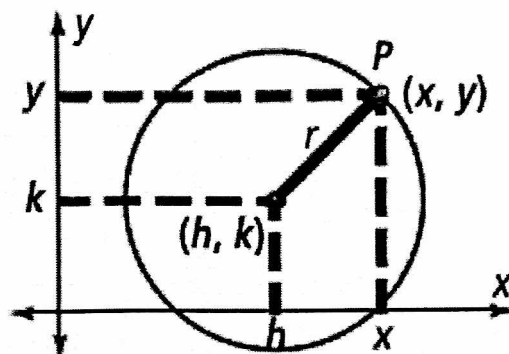$\sqrt{45} = \sqrt{9} \cdot \sqrt{5}$

$3\sqrt{5}$

6.10 Equations of Circles

SWBAT graph circles on the coordinate plane and write the equations of circles in standard form.

Standard Form of Circles

$$(x-h)^2 + (y-k)^2 = r^2$$

Center: (h,k)

Radius: r

Point on the circle: (x,y)

Example 1: Write the equation of a circle with the given information.

a) Center (0,0), Radius=10

$h = 0 \qquad k = 0 \qquad r = 10$

$(x-0)^2 + (y-0)^2 = 10^2$

$x^2 + y^2 = 100$

b) Center (2, 3), Diameter=12   ← *not radius!*

$h = 2 \qquad k = 3 \qquad r = \cancel{12}\, 6$

$(x-2)^2 + (y-3)^2 = 36$

Example 2: Determine the center and radius of a circle the given equation.

a) $x^2 + y^2 = \dfrac{9}{4}$

center: $(0,0)$

radius: $\sqrt{\dfrac{9}{4}} = \dfrac{\sqrt{9}}{\sqrt{4}} = \dfrac{3}{2}$ or 1.5

b) $(x+3)^2 + (y-5)^2 = 81$
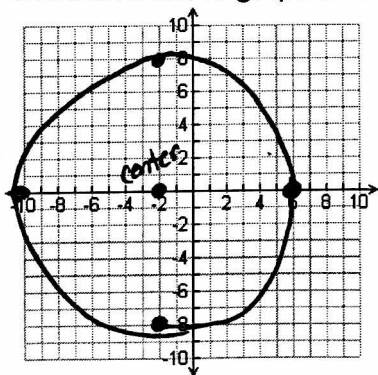
center: $(-3,5)$

radius: 9

c) $(x+4)^2 + (y+6)^2 = 1$

$c: (-4,-6)$

$r = 1$

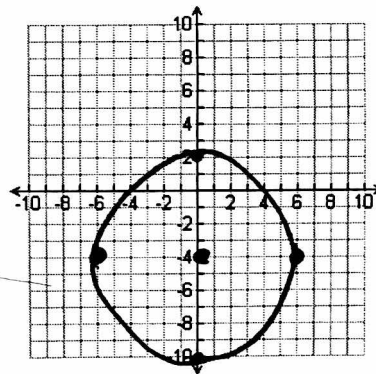**Example 3:** Use the center and the radius to graph each circle.

a) $(x+2)^2 + y^2 = 64$

Center: $(-2, 0)$

Radius: 8

b) $x^2 + (y+4)^2 = 36$

Center: $(0, -4)$

Radius: 6

Writing an Equation with a Pass-Thru Point

Step 1: Substitute the center (h, k) into the equation

Step 2: Substitute the "pass through point (x, y)" into the equation for x and y.

Step 3: Simplify and solve for r².

Step 4: Substitute r² back into the equation from Step 1.

Example 4: Write the equation of a circle with a given center (2, 5) that passes through the point (5 ,-1).

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(5-2)^2 + (-1-5)^2 = r^2$$

$$(3)^2 + (-6)^2 = r^2$$

$$9 + 36 = r^2$$

$$45 = r^2 \quad \text{stop here for eqn.}$$

$$(x-2)^2 + (y-5)^2 = 45$$