

Precalculus- Mini Unit Sequences and Series HW

Day 1 Homework

Directions 1-4: Determine whether the following sequences are arithmetic or not. If it is arithmetic, find the common difference.

1. 5, -2, -9, -16... arithmetic
 $\begin{array}{c} \downarrow \downarrow \downarrow \\ -7 -7 -7 \\ d = -7 \end{array}$

2. $\frac{1}{2}, 4, \frac{15}{2}, 11, \dots$ arithmetic
 $\begin{array}{c} \downarrow \downarrow \downarrow \\ +\frac{7}{2} +\frac{7}{2} +\frac{7}{2} \\ d = \frac{7}{2} \end{array}$

3. 1, 4, 9, 16... not arithmetic
 $\begin{array}{c} \downarrow \downarrow \downarrow \\ +3 +5 +7 \end{array}$

4. 29, 25, 21, 17, 13, 9 arithmetic
 $\begin{array}{c} \downarrow \downarrow \downarrow \downarrow \downarrow \\ -4 -4 -4 -4 -4 \\ d = -4 \end{array}$

5. What is the value of the first term in the arithmetic sequences if $a_6 = 87$ and $a_{12} = 129$?

$$m = \frac{129 - 87}{12 - 6} = \frac{42}{6} = 7$$

$$a_n = a_1 + d(n-1)$$

$$87 = a_1 + 7(6-1) \quad \boxed{a_1 = 52}$$

$$87 = a_1 + 7(5)$$

6. Write the first five terms of the sequence $a_n = \frac{(-1)^n}{(2n+1)!}$.

$$a_1 = \frac{(-1)^1}{(2+1)!} = \frac{-1}{3 \cdot 2 \cdot 1} = \boxed{\frac{-1}{6}}$$

$$a_2 = \frac{(-1)^2}{(4+1)!} = \frac{1}{5!} = \boxed{\frac{1}{120}}$$

$$a_3 = \frac{(-1)^3}{(6+1)!} = \frac{-1}{7!} = \boxed{\frac{-1}{5040}}$$

$$a_4 = \frac{(-1)^4}{(8+1)!} = \frac{1}{9!} = \boxed{\frac{1}{362,880}}$$

$$a_5 = \frac{(-1)^5}{(10+1)!} = \frac{-1}{11!} = \boxed{\frac{-1}{39,916,800}}$$

7. Write the first five terms of the sequence defined recursively

$$a_1 = 9$$

$$a_{k+1} = a_k - 4$$

$$a_1 = 9$$

$$a_2 = a_1 - 4 = 9 - 4 = \boxed{5}$$

$$a_4 = a_3 - 4 = 1 - 4 = \boxed{-3}$$

$$a_3 = a_2 - 4 = 5 - 4 = \boxed{1}$$

$$a_5 = a_4 - 4 = -3 - 4 = \boxed{-7}$$

8. Simplify the factorial expression $\frac{(2n-1)!}{(2n+1)!}$

$$\frac{(2n-1)!}{(2n+1)!} = \frac{\cancel{(2n-1)} \cancel{(2n-2)} \cancel{(2n-3)} \dots \cancel{3} \cancel{2} \cancel{1}}{(2n+1)(2n)\cancel{(2n-1)}\cancel{(2n-2)} \dots \cancel{3} \cancel{2} \cancel{1}} = \boxed{\frac{1}{(2n+1)(2n)}} \text{ or } \boxed{\frac{1}{4n^2 + 2n}}$$

9. Find the partial sum $\sum_{n=3}^{80} 5n$

$$n = 80 - 3 + 1 = 78$$

$$a_1 = 5(3) = 15$$

$$a_{80} = 5(80) = 400$$

$$S_n = \frac{n}{2} (a_1 + a_n)$$

$$S_{78} = \frac{78}{2} (15 + 400) = 39(415) = \boxed{16,185}$$

10. Given the series $-12+2+16+\dots+506$, determine how many terms are being added; then find the sum.

$$a_n = a_1 + d(n-1) \quad d = 14$$

$$506 = -12 + 14(n-1)$$

$$518 = 14(n-1)$$

$$37 = n-1$$

$$\boxed{38 = n}$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_{38} = \frac{38}{2}(-12 + 506)$$

$$= 19(494) = \boxed{9386}$$

11. Write the formula for the n th term of the arithmetic sequence if $a_4 = -10$ and $a_{10} = -25$.

$$m = \frac{-25 - (-10)}{10 - 4} = \frac{-15}{6} = -\frac{5}{2}$$

$$(4, -10) \quad (10, -25)$$

$$y = mx + b$$

$$-10 = -\frac{5}{2}(4) + b$$

$$-10 = -10 + b \quad b = 0$$

$$\boxed{a_n = -\frac{5}{2}n}$$