Day 2 Homework

Directions 1-4: Determine whether the following sequences are arithmetic, geometric, or neither. If it is arithmetic, find the common difference. If it is geometric, find the common ratio.

1. 40, 20, 10, 5…
   $\cdot\frac{1}{2}$ $\cdot\frac{1}{2}$ $\cdot\frac{1}{2}$
   geometric
   $r=\frac{1}{2}$

2. 2, -4, 8, -16, 32…
   $\cdot -2$ $\cdot -2$ $\cdot -2$ $\cdot -2$
   geometric
   $r=-2$

3. 1, 2, 3, 4, 5…
   +1 +1 +1
   arithmetic
   $d=1$

4. 1, 1, 2, 3, 5, 8, 13…
   +0 +1 +1
   $\cdot 1$ $\cdot 2$ $\cdot\frac{3}{2}$
   neither

5. Find the sum of the first 100 positive multiples of 5.

$5, 10, 15, 20, \ldots a_{100}$ $\quad d=5$

$a_n = a_1 + d(n-1)$

$a_{100} = 5+5(100-1) = 500$

$S_n = \frac{n}{2}(a_1+a_n)$

$S_{100} = \frac{100}{2}(5+500)$

$\boxed{S_{100} = 25,250}$

6. Find the first five terms of the geometric sequences if $a_1 = 9$ and $a_3 = 4$.

$a_n = a_1(r)^{n-1}$

$a_3 = 9(r)^{3-1}$

$4 = 9(r)^2$

$4/9 = r^2$

$\pm 2/3 = r$

$a_n = 9(\frac{2}{3})^{n-1}$

$\boxed{9, 6, 4, 8/3, 16/9}$

OR

$a_n = 9(-\frac{2}{3})^{n-1}$

$\boxed{9, -6, 4, -8/3, 16/9}$

7. Simplify the factorial $\frac{513!}{510!}$

$\frac{513!}{510!} = \frac{513\cdot 512\cdot 511\cdot \cancel{510!}}{\cancel{510!}} = 134,217,216$

8. Find the sum $\sum_{i=1}^{5}(3)^{i-1}$.

$n = 5-1+1 = 5$

$a_1 = 1$

$r = 3$

$S_5 = 1\left(\frac{1-3^5}{1-3}\right) = \boxed{121}$

9. Use sigma (summation) notation to write the sum: 10+15+20+25

$\boxed{\sum_{n=1}^{4} 5n-5}$

$a_n = a_1 + d(n-1)$

$a_n = 10+5(n-1)$

$a_n = 5n-5$

10. Find the sum of the first 120 terms of the arithmetic sequences with the given characteristics. $a_1 = 12$ and $d = 3$.

$a_n = a_1 + d(n-1)$

$a_{120} = 12+3(120-1)$

$a_{120} = 12+3(119)$

$a_{120} = 369$

$S_n = \frac{n}{2}(a_1+a_n)$

$S_{120} = \frac{120}{2}(12+369)$

$\boxed{S_{120} = 22860}$

11. Find the nth term of the geometric sequence. Then find the sum of the first 20 terms. Round to 3 decimal places. $a_1 = 16$ and $a_2 = -8$

$r = -\frac{1}{2}$

$a_n = a_1(r)^{n-1}$

$a_n = 16\left(-\frac{1}{2}\right)^{n-1}$

$S_{20} = 16\left(\frac{1-\left(-\frac{1}{2}\right)^{20}}{1-\left(-\frac{1}{2}\right)}\right)$

$\boxed{S_{20} = 10.667}$