Pre-Calculus Objective 1.02 Conics

1. According to Kepler's first law of planetary motion, each planet moves in an ellipse with the sun at one focus. Assume that one focus (the Sun) has coordinates (0,0) and the major axis of each planetary ellipse is the x-axis on a cosmic coordinate system (one unit = one billion kilometers). The minimum and maximum distances for Neptune are 4.456 and 4.537 billion kilometers, respectively. And the minimum and maximum distances for Pluto are 4.425 and 7.375 billion kilometers, respectively.
   a) For each planet determine the coordinates of the center and second focus.

   b) Write an equation that represents the orbit. Determine the eccentricity.

2. Suppose a satellite is in an elliptical orbit with the center of the Earth as one of its foci. The orbit has a major axis of 8910 miles with a minor axis of 8800 miles.
   a) Write an equation to model the path of the satellite.

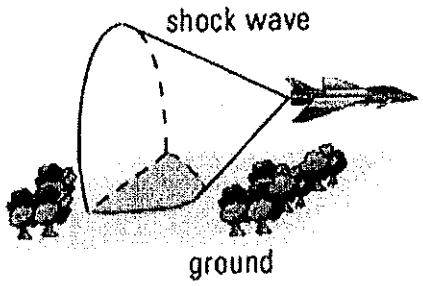
   b) How far is the Earth from the center of the elliptical path?

3. A parabolic satellite dish is modeled by the equation $y = \frac{1}{12}x^2$ and is measured in feet. In order to receive optimal signals, a satellite company must construct the receiver to be the focus of the parabolic dish. How far from the vertex of the dish should the receiver be placed?

4. Given the following equation: $4y^2 - 2x - 16y = -13 - x^2$
   a) Describe the type of conic section that is represented by the equation. Justify your response.

   b) Write the equation in standard form and sketch the graph that models the equation of the conic section.

5. When an airplane travels faster than the speed of sound, the sound waves form a cone behind the airplane. If the airplane is flying parallel to the ground, the sound waves intersect the ground in a hyperbola with the airplane directly above its center. A sonic boom is heard along a hyperbola. If you hear a sonic boom that is audible along a hyperbola with the equation $\frac{x^2}{100} - \frac{y^2}{4} = 1$ where x and y are measured in miles.
What is the shortest horizontal distance you could be to the airplane?



6. The shape of a roller coaster loop in an amusement park can be modeled by $\frac{y^2}{4225} + \frac{x^2}{2500} = 1$ where x and y are measured in feet.

   a) What is the width of the loop along the horizontal axis?

   b) Determine the height of the roller coaster from the ground when it reaches the top of the loop, if the lower rail is 25 feet from ground level.

Pre-Calculus Objective 1.02 Conics

(0,0) (1.642,0)
F C F

1. According to Kepler's first law of planetary motion, each planet moves in an ellipse with the sun at one focus. Assume that one focus (the Sun) has coordinates (0,0) and the major axis of each planetary ellipse is the x-axis on a cosmic coordinate system (one unit = one billion kilometers). The minimum and maximum distances for Neptune are 4.456 and 4.537 billion kilometers, respectively. And the minimum and maximum distances for Pluto are 4.425 and 7.375 billion kilometers, respectively.

Neptune
a = 2.2685
b = 2.228
c = 1.642

Pluto
a = 3.6875
b = 2.2125
c = 2.95

   a) For each planet determine the coordinates of the center and second focus.

Neptune Focus → (3.284, 0) center → (1.642, 0)

Pluto Focus → (5.9, 0) center → (2.95, 0)

   b) Write an equation that represents the orbit. Determine the eccentricity.

Neptune $\frac{(x-1.642)^2}{2.2685^2} + \frac{y^2}{2.228^2} = 1$ $e = \frac{1.642}{2.2685} = .724$

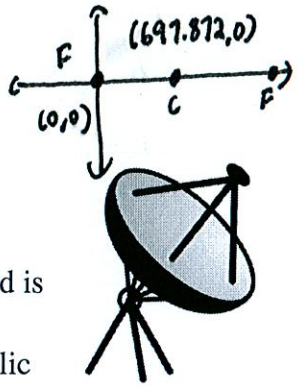
Pluto $\frac{(x-2.95)^2}{3.6875^2} + \frac{y^2}{2.2125^2} = 1$ $e = \frac{2.95}{3.6875}$ $e = .8$

2. Suppose a satellite is in an elliptical orbit with the center of the Earth as one of its foci. The orbit has a major axis of 8910 miles with a minor axis of 8800 miles.

a = 4455
b = 4400
c = 697.872

   a) Write an equation to model the path of the satellite.

$\frac{(x-697.872)^2}{4455^2} + \frac{y^2}{4400^2} = 1$



   b) How far is the Earth from the center of the elliptical path?

697.872 miles

3. A parabolic satellite dish is modeled by the equation $y = \frac{1}{12}x^2$ and is measured in feet. In order to receive optimal signals, a satellite company must construct the receiver to be the focus of the parabolic dish. How far from the vertex of the dish should the receiver be placed?

$\frac{1}{4a} = \frac{1}{4(\frac{1}{12})} = \frac{1}{\frac{1}{3}} = 3$ 3 feet

$a = \frac{1}{12}$

4. Given the following equation: $4y^2 - 2x - 16y = -13 - x^2$

   a) Describe the type of conic section that is represented by the equation. Justify your response.

$x^2 - 2x + 4y^2 - 16y = -13$ ELLIPSE

x^2 and y^2 have the same sign but different coefficients

   b) Write the equation in standard form and sketch the graph that models the equation of the conic section.

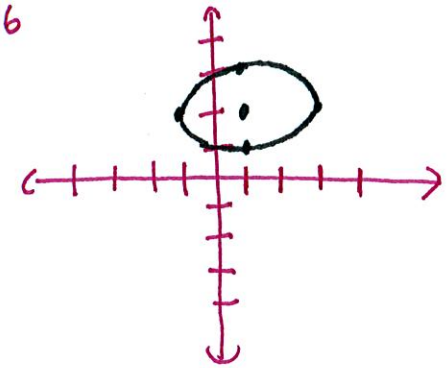
$x^2 - 2x + 1 + 4(y^2 - 4y + 4) = -13 + 1 + 16$

$\frac{(x-1)^2}{4} + \frac{4(y-2)^2}{4} = \frac{4}{4}$

$\frac{(x-1)^2}{4} + \frac{(y-2)^2}{1} = 1$

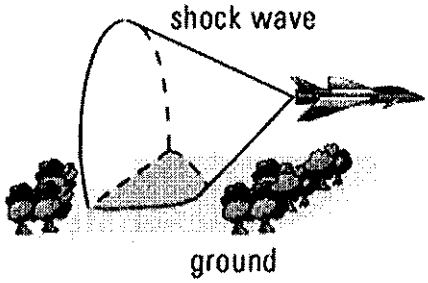
a = 2 b = 1

center → (1, 2)



5. When an airplane travels faster than the speed of sound, the sound waves form a cone behind the airplane. If the airplane is flying parallel to the ground, the sound waves intersect the ground in a hyperbola with the airplane directly above its center. A sonic boom is heard along a hyperbola. If you hear a sonic boom that is audible along a hyperbola with the equation $\frac{x^2}{100} - \frac{y^2}{4} = 1$ where x and y are measured in miles. $a=10$ $b=2$ What is the shortest horizontal distance you could be to the airplane?

10 miles



6. The shape of a roller coaster loop in an amusement park can be modeled by $\frac{y^2}{4225} + \frac{x^2}{2500} = 1$ where x and y are measured in feet.

$a=65$ $b=50$

a) What is the width of the loop along the horizontal axis?

100 ft.

b) Determine the height of the roller coaster from the ground when it reaches the top of the loop, if the lower rail is 25 feet from ground level.

$65 \times 2 = 130 + 25$

155 ft.