Pre-Calculus Objective 2.05 Polar Equations/Objective 2.06 Parametric Equations

1. When recording live performances, sound engineers often use a cardioid microphone because it captures the singer's voice with limited outside noise from the audience. Suppose the boundary of the optimal pickup region is given by the equation $r = 2 + 2\sin\theta$, where r is measured in meters from the microphone on the mic stand. What is the maximum distance a musician could stand away from the microphone and still be within this boundary?

2. Find the intersection of the following two polar graphs, without using a calculator.

$$r = 2 + 3\sin\theta \text{ and } r = \sin\theta$$

3. Archaeologists want to create a map of a recent dig using polar coordinates. On their grid, they used a rectangular coordinate system and marked the king's tomb at $(-12, 5)$. What would be the new coordinates of the king's tomb if it were marked on a polar grid in degrees?

4. Convert the following equation from polar to rectangular form.

$$r = 6\cos\theta$$

5. Convert the following equation from rectangular to polar form.

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

6. Given the equations below:
   1) $4x^2 + 8y^2 - 8x + 48y + 44 = 0$
   2) $y^2 - 12x + 18y + 153 = 0$

   Identify the conic and write each equation in standard form.

7. Two rockets are fired from a space station. The first rocket's path can be described using the parametric equations $x(t) = t + 4$ and $y(t) = 3t - 1$. The second rocket's path can be described using the parametric equations $x(t) = t + 4$ and $y(t) = 2t + 9$.

a. Eliminate the parameter for the equations given for the first rocket and express its path as a function.

b. Eliminate the parameter for the equations given for the second rocket and express its path as a function.

c. Will the two rockets collide? Defend your answer using mathematical reasoning.

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1. When recording live performances, sound engineers often use a cardioid microphone because it captures the singer's voice with limited outside noise from the audience. Suppose the boundary of the optimal pickup region is given by the equation $r = 2 + 2\sin\theta$, where r is measured in meters from the microphone on the mic stand. What is the maximum distance a musician could stand away from the microphone and still be within this boundary?

$r = 2+2\sin\theta$ → 1 between → -1

$r = 2+2(1)$

4 meters

2. Find the intersection of the following two polar graphs, without using a calculator.

$r = 2 + 3\sin\theta$ and $r = \sin\theta$

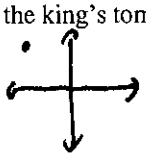
$2+3\sin\theta = \sin\theta$

$2 = -2\sin\theta$

$-1 = \sin\theta$

$\theta = \frac{3\pi}{2}$

3. Archaeologists want to create a map of a recent dig using polar coordinates. On their grid, they used a rectangular coordinate system and marked the king's tomb at $(-12, 5)$. What would be the new coordinates of the king's tomb if it were marked on a polar grid in degrees?



$r^2 = x^2+y^2$

$r^2 = (-12)^2+5^2$

$r = \pm 13$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{5}{-12}$

$\theta = -22.620^\circ + 180^\circ$

$\theta = 157.380^\circ$

$(13, 157.380^\circ)$

4. Convert the following equation from polar to rectangular form.

$r = 6\cos\theta$

LOOK AT GRAPH

$(x-3)^2+y^2 = 9$

5. Convert the following equation from rectangular to polar form.

$16x^2+25y^2 = 400$

$16(r\cos\theta)^2+25(r\sin\theta)^2 = 400$

$16r^2\cos^2\theta + 25r^2\sin^2\theta = 400$

$r^2(16\cos^2\theta+25\sin^2\theta) = 400$

$\frac{400}{400}\left(\frac{x^2}{25}+\frac{y^2}{16}\right) = (1)\frac{400}{}$

$r^2 = \frac{400}{(16\cos^2\theta+25\sin^2\theta)}$

OR $r = \pm\sqrt{\frac{400}{16\cos^2\theta+25\sin^2\theta}}$

6. Given the equations below:

1) $4x^2 + 8y^2 - 8x + 48y + 44 = 0$ ELLIPSE

2) $y^2 - 12x + 18y + 153 = 0$ PARABOLA

Identify the conic and write each ~~equation in standard~~ form.

$4x^2-8x+8y^2+48y = -44$

$4(x^2-2x+1)+8(y^2+6y+9) = -44+4+72$

$\frac{4(x-1)^2}{32}+\frac{8(y+3)^2}{32} = \frac{32}{32}$

$\frac{(x-1)^2}{8}+\frac{(y+3)^2}{4} = 1$

$y^2+18y+81+153-81 = 12x$

$\frac{(y+9)^2}{12}+\frac{72}{12} = \frac{12x}{12}$

$x = \frac{1}{12}(y+9)^2+6$

7. Two rockets are fired from a space station. The first rocket's path can be described using the parametric equations $x(t) = t + 4$ and $y(t) = 3t - 1$. The second rocket's path can be described using the parametric equations $x(t) = t + 4$ and $y(t) = 2t + 9$.

a. Eliminate the parameter for the equations given for the first rocket and express its path as a function.

$t = x-4$ $\quad y = 3(x-4)-1$

$y = 3x-13$

b. Eliminate the parameter for the equations given for the second rocket and express its path as a function.

$t = x-4$ $\quad y = 2(x-4)+9$

$y = 2x+1$

c. Will the two rockets collide? Defend your answer using mathematical reasoning.

$3x-13 = 2x+1$

$x = 14$

yes when $x=14$ or $t=10$