

Pre-Calculus Objective 2.08 Limits of Functions

1. The value of the expression $200 \left(1 + \frac{0.05}{k}\right)^{10k}$ gives the account balance if \$200 is invested for 10 years in an account that pays 5% annual interest that is compounded k times a year.

a) Find the account balance if $k = 4, 12, 52, 356, 8760,$ or $525,600$ compounded for each quarter, month, week, day, hour, minute.

b) What value does $200 \left(1 + \frac{0.05}{k}\right)^{10k}$ approach as k approaches infinity?

2. Let $f(x) = \frac{x+1}{(x-2)(x+3)}$. Use the table of values to determine what happens to $f(x)$ as $x \rightarrow -3$.

3. Describe the behavior of $f(x) = \frac{1}{x} \sin(x)$ for large, positive values of x .

4. Describe the behavior of $f(x) = \sin\left(\frac{\pi}{x}\right)$ as $x \rightarrow 0$.

5. Find the following limits without using a calculator:

a) $\lim_{x \rightarrow 2} \frac{2x^2 - 4x}{x^2 - 4}$

b) $\lim_{x \rightarrow 5} \frac{\frac{1}{x} - \frac{1}{5}}{x - 5}$

c) $\lim_{x \rightarrow \infty} \frac{4x^2 - 5x}{3x^2 + 4}$

6. What value of n would make the function continuous? Show both graphically and algebraically.

$$f(x) = \begin{cases} x^2 + 1 & x < -1 \\ n & -1 \leq x \leq 3 \\ 5 - x & x > 3 \end{cases}$$

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1. The value of the expression $200 \left(1 + \frac{0.05}{k}\right)^{10k}$ gives the account balance if \$200 is invested for 10 years in an account that pays 5% annual interest that is compounded k times a year.

a) Find the account balance if $k = 4, 12, 52, 356, 8760,$ or $525,600$ compounded for each quarter, month, week, day, hour, minute.

$$\begin{array}{lll} k=4 & \$326.72 & k=52 & \$329.67 & k=8760 & \$329.74 \\ k=12 & \$329.40 & k=356 & \$329.73 & k=525,600 & \$329.74 \end{array}$$

b) What value does $200 \left(1 + \frac{0.05}{k}\right)^{10k}$ approach as k approaches infinity?

$$\$329.74$$

2. Let $f(x) = \frac{x+1}{(x-2)(x+3)}$. Use the table of values to determine what happens to $f(x)$ as $x \rightarrow -3$.

$$\lim_{x \rightarrow 3^-} f(x) = -\infty \qquad \lim_{x \rightarrow 3^+} f(x) = \infty$$

3. Describe the behavior of $f(x) = \frac{1}{x} \sin(x)$ for large, positive values of x .

$$\lim_{x \rightarrow \infty} \frac{1}{x} \sin x = 0$$

4. Describe the behavior of $f(x) = \sin\left(\frac{\pi}{x}\right)$ as $x \rightarrow 0$.

$$\lim_{x \rightarrow 0} \sin\left(\frac{\pi}{x}\right) = 0$$

5. Find the following limits without using a calculator:

a) $\lim_{x \rightarrow 2} \frac{2x^2 - 4x}{x^2 - 4} = \lim_{x \rightarrow 2} \frac{2x(x-2)}{(x-2)(x+2)} = \frac{4}{4} = \boxed{1}$

b) $\lim_{x \rightarrow 5} \frac{\frac{5}{x} - \frac{1}{x-5}}{\frac{x}{x-5}} = \lim_{x \rightarrow 5} \frac{\frac{5-x}{5x}}{\frac{x}{x-5}} = \lim_{x \rightarrow 5} \frac{5-x}{5x} \cdot \frac{x-5}{x-5} = \lim_{x \rightarrow 5} \frac{-1(x-5)}{5x} \cdot \frac{1}{x-5} = \frac{-1}{25} = \boxed{\frac{-1}{25}}$

c) $\lim_{x \rightarrow \infty} \frac{2x^2 - 5x}{3x^2 + 4} = \boxed{\frac{2}{3}}$

6. What value of n would make the function continuous? Show both graphically and algebraically.

$$f(x) = \begin{cases} x^2 + 1 & x < -1 \\ n & -1 \leq x \leq 3 \\ 5 - x & x > 3 \end{cases}$$

$$\boxed{n=2}$$

$$\begin{aligned} (-1)^2 + 1 &= 2 \\ n &= 2 \end{aligned}$$

from left

$$\begin{aligned} 5 - 3 &= 2 \\ n &= 2 \end{aligned}$$

from right

needs to be the same value