Jumping Frogs

Experiments in a factorial situation. Estimating the effects of each factor and of the interaction.

Even the most consistent athletes do not perform exactly the same every time they play. We reason that their performance is affected by several factors, such as the strength of their opponents, their own schedules, or the state of their health. We can use statistics to estimate the effects of such factors on the outcomes. Our sports personality for this activity is going to be a paper frog, and we are interested in the distance it can jump. In particular, we want to find out how the weight and the size of the frog affect "jumping" distance.

We can carry out an experiment to measure the effects of these two factors. The directions for constructing the frogs follow this activity. We have a choice of the type and size of paper to use. We will use two sizes of paper and paper of two different weights. Determining the properties of the ultimate jumping frog is far more complicated and is thus beyond the scope of this activity.

Question

How can we measure one factor's effect on the outcome when we have two factors that can influence the outcome of an experiment?

Objectives

In this activity, you will learn how to analyze an experiment in which the outcomes are affected by two factors, each factor taking on two values. You will learn how to estimate the effect of each factor and the effect of the interaction of the factors on the outcome, without using a formal design and analysis of experiments approach.

Prerequisites

You should be familiar with stem and leaf diagrams and with the use of sample means as estimators.

Activity

1. We want to see how two factors, the size and the weight of the frog, influence "jumping" distance. In this activity, we will use only two sizes and two weights (that is, we will use two levels of each factor):

$$S = \text{size of the frog (2 levels = 2 sizes)}$$
$$W = \text{weight of the paper (2 levels = 2 weights)}$$

You will use the data on the distances the frogs jump to determine how size and weight affect the frog's jump and use these results to guide you in coming up with the champion frog.

(Note: Bigger frogs will naturally weigh more than smaller ones. But in this experiment, the term *weight* refers only to the "weight" of the paper. You can have small frogs made of heavy paper that may, in fact, weigh less than large frogs made of light paper.)

2. You will need square sheets of paper to construct your frog. Your instructor will have 7.5-inch and 6-inch squares in two weights of paper. Each of you will be randomly assigned one of the squares. Make a frog with your sheet of paper. Practice with your frog five times and report to the instructor the length of the last jump. (Measure the jump in centimeters.)

Enter the data for the class in Table 1.

		Size	
		Large	**Small**
Weight of Paper	**Heavy**		
	Light		

Table 1: Jumping distance

3. We will first look at the results of our experiments graphically.
   a. Construct a back-to-back stem and leaf diagram with all measurements for the large paper frog jumps on one side and those for the small paper frog jumps on the other side. Note that you are using both heavy and light paper measurements on each side. You may wish to show them with different symbols.
   b. Repeat the back-to-back stem and leaf diagram with heavy paper frog jumps on one side and light paper frog jumps on the other side. Again, you could indicate the large paper and small paper frog jumps with different symbols.
   c. What do the stem and leaf diagrams tell us about the relative distances jumped by large and small frogs? Are there differences in the patterns of jumps of the heavy and light frogs in the first back-to-back stem and leaf diagram?

d. Now turn your attention to the second back-to-back stem and leaf diagram. Compare the distances jumped by the heavy and light frogs in the second stem and leaf diagram. Does the size of the frog seem to affect the jump?

e. Combine your analyses to reach a conclusion on which factor has more of an effect on the jumps: size or weight.

4. We will now construct arithmetic summaries for the data. We will try to get an estimate of the effect of size and weight on a frog's jump. Let's look at the following table of means, Table 2.

		Size	
		Large (+)	**Small (−)**
Weight of Paper	**Heavy (+)**	Mean of heavy/large frogs = W_+S_+	Mean of heavy/small frogs = W_+S_-
	Light (−)	Mean of light/large frogs = W_-S_+	Mean of light/small frogs = W_-S_-

Table 2: Mean jumping distance

You now need to calculate the mean of all the heavy frogs and that of all the light frogs by getting the two row averages. So, for example, the mean jump for the heavy frogs is $1/2[(W_+S_+) + (W_+S_-)]$. Based on these averages, which weight gives better jumping frogs? To estimate the difference in the length of the jumps due to the different weights, calculate

$$A = 1/2(\text{mean jump of the heavy frogs} - \text{mean jump of the light frogs})$$

We will use this as an estimate of the effect of the weight of the paper. Now calculate the mean of all the large frogs and that of all the small frogs by calculating the column averages. Which size frog jumps better? The effect of the paper size is given by

$$B = 1/2(\text{mean jump of the large frogs} - \text{mean jump of the small frogs})$$

In the previous section, you reached a conclusion on the relative effects of weight and size. Do the values of A and B confirm it? Using the values of A and B, complete the following statements:

Changing the weight of the paper (frogs) from light to heavy (increases/decreases) the average distance jumped by ___________.

Changing the size of the paper (frogs) from small to large (increases/decreases) the average distance jumped by ___________.

5. Do the two factors interact with each other? In other words, is the difference in the mean jumps of the large frogs and that of small frogs different for the heavy paper and the light paper? We can estimate the interaction term as follows. First, for the large paper frogs, we calculate the average difference due to weight as

$$D1 = 1/2(\text{mean of the large/heavy} - \text{mean of the large/light})$$

and for the small frogs we get

$$D2 = 1/2(\text{mean of the small/heavy} - \text{mean of the small/light})$$

The quantity $1/2(D1 - D2)$ gives us a measure of the interaction term or the effect of changing the levels of both factors. Looking at the table of means and the estimate of the interaction between the two factors, are the differences in the average jumps between large and small frogs the same for both weights of paper?

Wrap-Up

1. We were able to estimate the effect of the two factors using means. Do the following graphical analysis: Find the overall mean. Complete the following graph using the row and column means from Table 2. Above the label "Size" you will plot the two column means, and plot the two row means above the label "Weight." Use the appropriate vertical scale.

   Look at the distances of the two means for size from the overall mean. Are the two means equidistant from the overall mean? Compare the magnitude of this distance to the value of *B* that you computed earlier. Do a similar analysis on the distances of the two means for weight from the overall mean, comparing them with the value of *A*. Does it seem reasonable to use *A* and *B* as measures of the effects of the two factors? Comment.

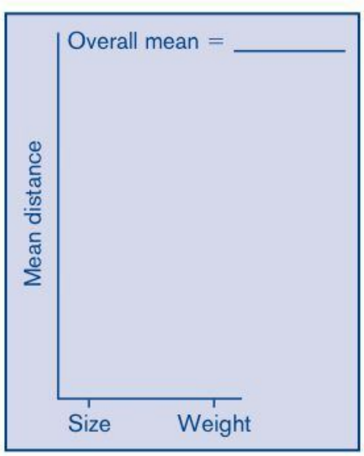

Figure 1

2. Complete the following graph using the means W_+S_+, W_-S_+, W_+S_-, and W_-S_- from Table 2. You will plot W_+S_+ and W_-S_+ above "Large," and W_+S_- and W_-S_- above "Small."

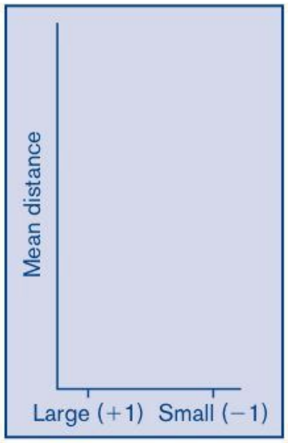

Figure 2

Now join the points W_+S_+ and W_+S_-. Also join the points W_-S_+ and W_-S_-. You now have two lines. Compare the slopes of the two lines. Are the lines parallel? If we replace "Large" and "Small" on the horizontal axis by $+1$ and -1, respectively, we can get the slopes of the two lines. Calculate the slopes and compare them with $D1$ and $D2$ that you calculated earlier. Explain how the measure for interaction of the two factors is related to the slopes of the two lines.

Extensions

This activity is an analysis of data from a designed experiment. The response was affected by two factors, the size and the weight of paper, each factor taking on two values. We estimated the effects of the size and the weight of paper used on the jumps of the frogs as well as the interaction of these two factors.

1. Your instructor randomly assigned a frog to you. Why was this done? Give examples of how nonrandom assignments could affect the analysis.
2. Write a brief report describing the design and analysis of an experiment to find out how test scores are affected by the time the test is administered and the level of difficulty of the test.

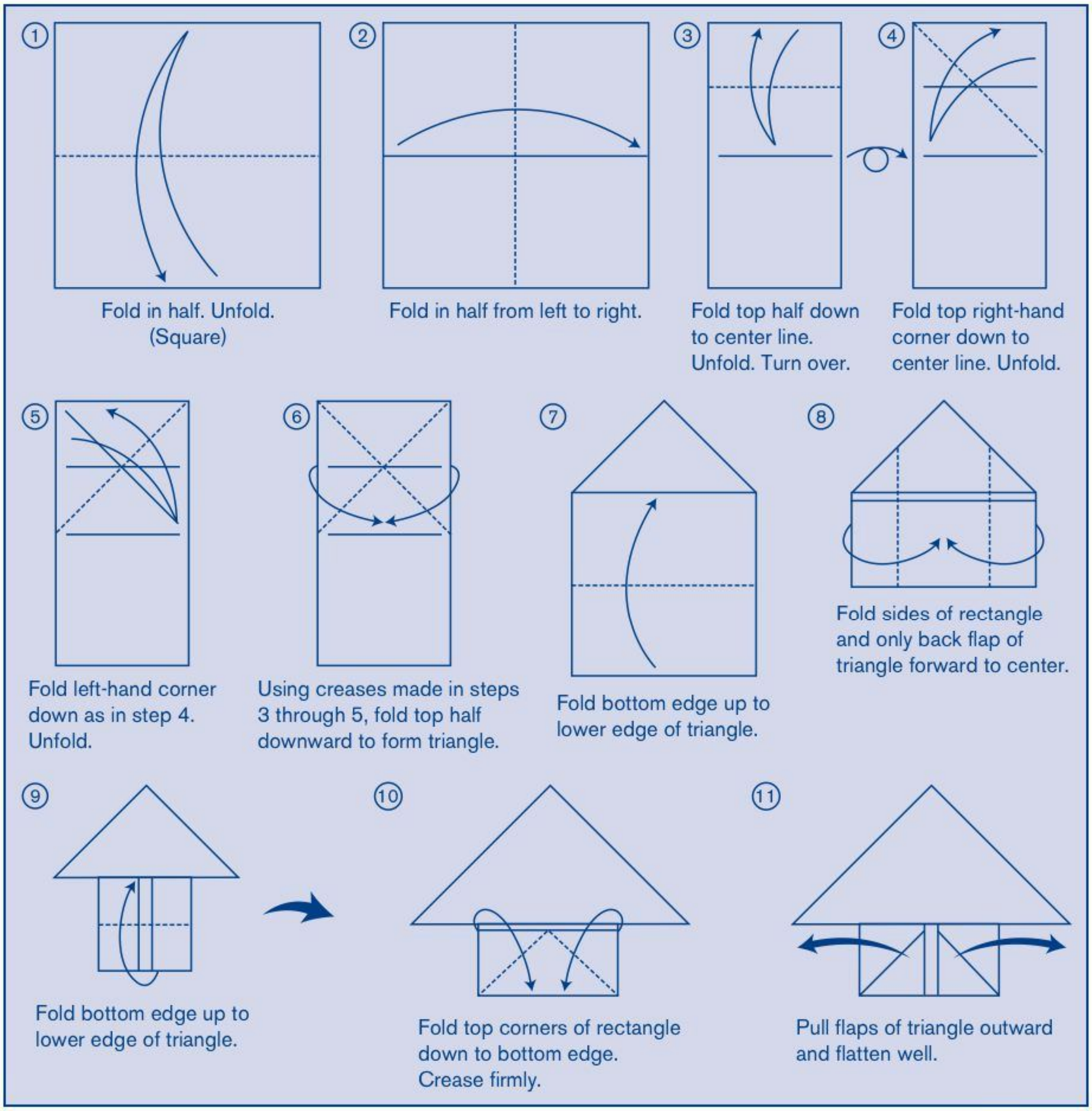

Figure 3 ***(continued)***

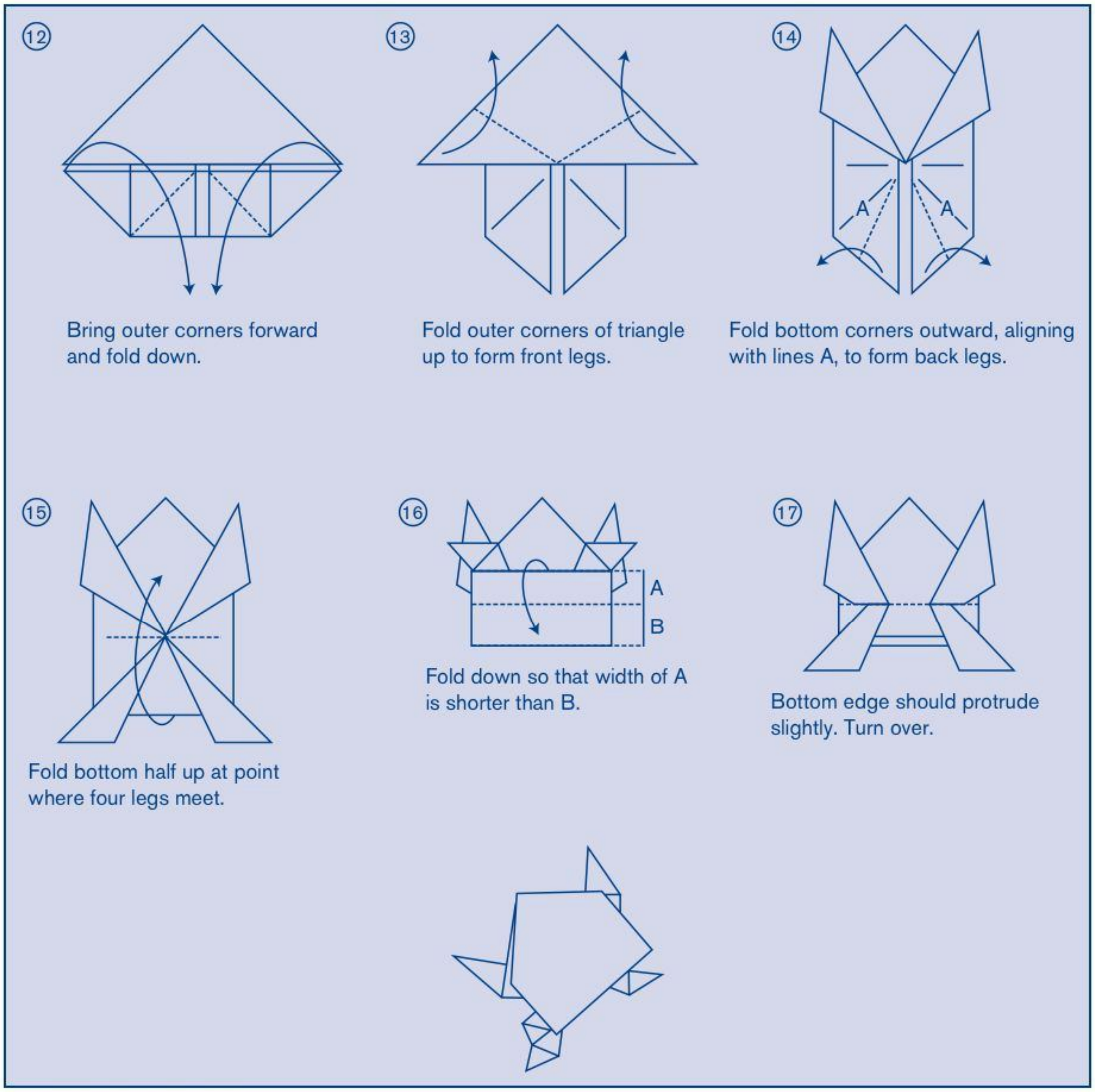

Figure 3 ***(continued)***

Assessment Questions

1. Why do you suppose the "effect" sizes *A* and *B* have a (1/2) in them?
2. Describe a real-life situation analogous to the frogs—two variables, two levels each—that would have a large interaction term.