Precalculus Name Key

5.2 Notes: Verifying Trigonometric Identities

Verifying Trigonometric Identities Algebraically: DO NOT CROSS THE EQUAL SIGN! Since you are trying to PROVE that both sides of the given equation are equal, you cannot use any properties of equality because that would mean that you already assume that they are equal.

Strategies:

- ***Work with the more complicated side of the equation and make it match the simpler side.***
- ***Simplify both sides until they match.***
- ***Look to factor.***
- ***Rewrite everything in terms of sine and cosine.***
- ***Look for a substitution.***
- ***Perform indicated operation (add, subtract, etc…)***
- ***Rationalizing***

Examples: Verify each identity algebraically.

1. $\dfrac{\sin^2 x - 1}{\sin^2 x} = -\cot^2 x$

*factor out -1

$\dfrac{-1(1-\sin^2 x)}{\sin^2 x} =$

$-\dfrac{\cos^2 x}{\sin^2 x} =$

$-\cot^2 x = -\cot x$

2. $\cot x \cdot \cos x = \dfrac{1}{\tan x \sec x}$

$= \dfrac{1}{\dfrac{\sin x}{\cos x} \cdot \dfrac{1}{\cos x}}$

$= \dfrac{\cos x}{\sin x} \cdot \dfrac{\cos x}{1}$

$\cot x \cdot \cos x = \cot x \cdot \cos x$

3. $\dfrac{\cot^2\theta}{1+\csc\theta} = \dfrac{1-\sin\theta}{\sin\theta}$

$\dfrac{(1-\csc\theta)}{(1-\csc\theta)} \cdot \dfrac{\cot^2\theta}{(1+\csc\theta)} =$

$\dfrac{(1-\csc\theta)\cot^2\theta}{1-\csc^2\theta} =$

$\dfrac{(1-\csc\theta)\cancel{\cot^2\theta}}{-\cancel{\cot^2\theta}} =$

$-1 + \dfrac{1}{\sin\theta} =$

$\dfrac{-\sin\theta + 1}{\sin\theta} \rightarrow \dfrac{1-\sin\theta}{\sin\theta} = \dfrac{1-\sin\theta}{\sin\theta}$

4. $1-\sin^4 x = \cos^2 x\left(2-\cos^2 x\right)$

$= (1-\sin^2 x)(2-(1-\sin^2 x))$

$= (1-\sin^2 x)(1+\sin^2 x)$

$1-\sin^4 x = 1-\sin^4 x$

#3 $\dfrac{\csc^2\theta - 1}{1+\csc\theta} = \dfrac{1}{\sin\theta} - \dfrac{\sin\theta}{\sin\theta}$

*Diff of squares

$\dfrac{(\cancel{\csc\theta+1})(\csc\theta-1)}{(\cancel{1+\csc\theta})} = \csc\theta - 1$

$\csc\theta - 1 = \csc\theta - 1$

* common denominator

5. $\frac{\sec^2\theta-1}{\sec^2\theta}=\sin^2\theta$ xsplit fraction $\frac{\tan^2\theta}{\sec^2\theta}=$ $\frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}}=$ $\frac{\sin^2\theta}{\cancel{\cos^2\theta}}\cdot\frac{\cancel{\cos^2\theta}}{1}=$ $\frac{\sec^2\theta}{\sec^2\theta}-\frac{1}{\sec^2\theta}$ $1-\cos^2\theta$ $\sin^2\theta$ $\sin^2\theta=\sin^2\theta$	**6.** $\frac{1}{1-\sin x}+\frac{1}{1+\sin x}=2\sec^2 x$ $\frac{1+\sin x+1-\sin x}{(1-\sin x)(1+\sin x)}=$ $\frac{2}{1-\sin^2 x}=$ $\frac{2}{\cos^2 x}=$ $2\sec^2 x=2\sec^2 x$
7. $\underbrace{\sec^2 y}-\tan^2 y=\tan y\cot y$ $(1+\tan^2 y)-\tan^2 y=\tan y\cdot\frac{1}{\tan y}$ $1=1$ $=\tan y\cot y$ $=\tan y\cdot\frac{1}{\tan y}$ $=1$	**8.** $\sec^4 A-\sec^2 A=\frac{1}{\cot^4 A}+\frac{1}{\cot^2 A}$ $=\tan^4 A+\tan^2 A$ $=\tan^2 A(\tan^2 A+1)$ $=(\sec^2 A-1)(\sec^2 A-1+1)$ $=(\sec^2 A-1)(\sec^2 A)$ $\sec^4 A-\sec^2 A=\sec^4 A-\sec^2 A$

To verify an identity graphically:	**To verify an identity numerically:**
▪ You want to show that each side of the equation has the same graph. ▪ Enter the left-hand side of the equation into y_1 and the right-hand side of the equation in y_2. ▪ When you look at the graphs they should appear to coincide.	▪ You want to show that for every value of x makes the equation true (makes both sides equal). ▪ To verify this, after typing in both sides of the equation in to your calculator, look at your table. The values for y_1 and y_2 should be the same for every x value.

Verify the identity graphically and numerically. Fill in the values of *y* for both equations.

9. $\sec^4 A-\sec^2 A=\frac{1}{\cot^4 A}+\frac{1}{\cot^2 A}$

$y_1=\sec^4 A-\sec^2 A=\frac{1}{\cos^4 A}-\frac{1}{\cos^2 A}$

$y_2=\frac{1}{\cot^4 A}+\frac{1}{\cot^2 A}=\tan^4 A+\tan^2 A$

	y_1	y_2
10	0.59708	0.59708
20	30.054	30.054
30	1724.347	1724.347