

Precalculus

Name Key

5.1 Notes: Using Fundamental Identities

Reciprocal Identities			Quotient Identities
$\sin \theta = \frac{1}{\csc \theta}$	$\cos \theta = \frac{1}{\sec \theta}$	$\tan \theta = \frac{1}{\cot \theta}$	$\tan \theta = \frac{\sin \theta}{\cos \theta}$
$\csc \theta = \frac{1}{\sin \theta}$	$\sec \theta = \frac{1}{\cos \theta}$	$\cot \theta = \frac{1}{\tan \theta}$	$\cot \theta = \frac{\cos \theta}{\sin \theta}$
Pythagorean Identities			$\therefore \text{by } \sin^2 \theta$
$\sin^2 \theta + \cos^2 \theta = 1$	$1 + \tan^2 \theta = \sec^2 \theta$ $1 = \sec^2 \theta - \tan^2 \theta$	$1 + \cot^2 \theta = \csc^2 \theta$ $1 = \csc^2 \theta - \cot^2 \theta$	$\tan^2 \theta = \sec^2 \theta - 1$ $\cot^2 \theta = \csc^2 \theta - 1$

Using Identities to Evaluate a Function

1. If $\cos u = -\frac{2}{3}$ and $\tan u > 0$ then find the values of all 6 trigonometric functions.	* Quadrant III
$\sin^2 \theta + \cos^2 \theta = 1$	$\sin u = -\frac{\sqrt{5}}{3}$
$\sin^2 \theta + (-\frac{2}{3})^2 = 1$	$\cos u = -\frac{2}{3}$
$\sin^2 \theta + \frac{4}{9} = 1$	$\csc u = \frac{1}{\sin u} = \frac{1}{-\frac{\sqrt{5}}{3}} = -\frac{3}{\sqrt{5}}$
$\sin^2 \theta = \frac{5}{9}$	$\sec u = \frac{1}{\cos u} = \frac{1}{-\frac{2}{3}} = -\frac{3}{2}$
$\sin \theta = \pm \frac{\sqrt{5}}{3}$	$\tan u = \frac{\sin u}{\cos u} = \frac{-\frac{\sqrt{5}}{3}}{-\frac{2}{3}} = \frac{\sqrt{5}}{2}$
2. If $\cot x = -5$ and $\sin x = \frac{\sqrt{26}}{26}$ then find the values of all 6 trigonometric functions.	* Quadrant II
$1 + \tan^2 x = \sec^2 x$	$\sin x = \frac{\sqrt{26}}{26}$
$1 + (-\frac{1}{5})^2 = \sec^2 x$	$\csc x = \sqrt{26}$
$1 + \frac{1}{25} = \sec^2 x$	$\frac{1}{\sec x} = \cos x = -\frac{5\sqrt{26}}{26}$
$\frac{26}{25} = \sec^2 x$	$\sec x = -\frac{\sqrt{26}}{5}$
$\pm \frac{\sqrt{26}}{5} = \sec x$	$\tan x = \frac{1}{\cot x} = -\frac{1}{5}$
	$\cot x = -5$

Simplifying a Trigonometric Expression

3. $\sin x \cos^2 x - \sin x$

$$\sin x (\cos^2 x - 1)$$

$$-\sin x (1 - \cos^2 x)$$

$$-\sin x (\sin^2 x)$$

$$-\sin^3 x$$

4. $\frac{\sec \theta}{\csc \theta}$

$$\frac{1}{\cos \theta}$$

$$\frac{1}{\sin \theta}$$

$$\frac{1}{\cos \theta} \cdot \frac{\sin \theta}{1}$$

$$\frac{\sin \theta}{\cos \theta}$$

$$\tan \theta$$

5. $\csc x - \cos x \cot x$

$$\frac{1}{\sin x} - \frac{\cos x}{1} \cdot \frac{\cos x}{\sin x}$$

$$\frac{1}{\sin x} - \frac{\cos^2 x}{\sin x}$$

$$\frac{1 - \cos^2 x}{\sin x}$$

$$\frac{\sin^2 x}{\sin x}$$

$$\sin x$$

6. $\frac{\csc \beta}{1 + \cot^2 \beta}$

$$\frac{\csc \beta}{\csc^2 \beta}$$

$$\frac{1}{\csc \beta}$$

$$\sin \beta$$

Factoring—Think of the trigonometric function as the variable.

7. $\cos^2 x - 1$

$$(\cos x + 1)(\cos x - 1)$$

8. $\sin^2 x - 4\sin x - 12$

$$(\sin x + 2)(\sin x - 6)$$

9. $\sec^2 x - \tan x - 3$

$$(1 + \tan^2 x) - \tan x - 3$$

$$\tan^2 x - \tan x - 2$$

$$(\tan x - 2)(\tan x + 1)$$